Sequence control of quadruped gaits using combinatorial threshold-linear networks

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Why quadruped gaits?

How does the brain generate diverse gaits, using a single circuit? How can gaits coexist in a single network? How can we quickly transition between gaits?

How does the brain learn and control motor sequences? How is the order of a sequence encoded in a network?

Combinatorial threshold-linear networks (CTLNs)

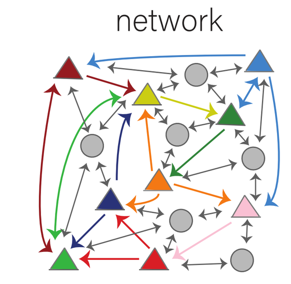
dynamics

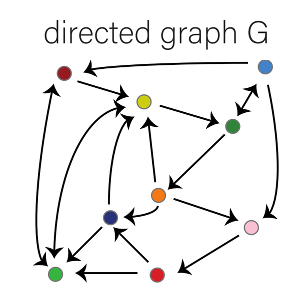
$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+$$

network model

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } j \to i \text{ in } G, \\ -1-\delta & \text{if } j \not\to i \text{ in } G. \end{cases}$$

$\delta > 0,\ 0 < \varepsilon < \frac{\delta}{\delta+1},\ \theta > 0$





in simulations shown here: $\varepsilon = 0.25, \delta = 0.5, \theta = 0.1.$

We label all the fixed points of a network by their **support**:

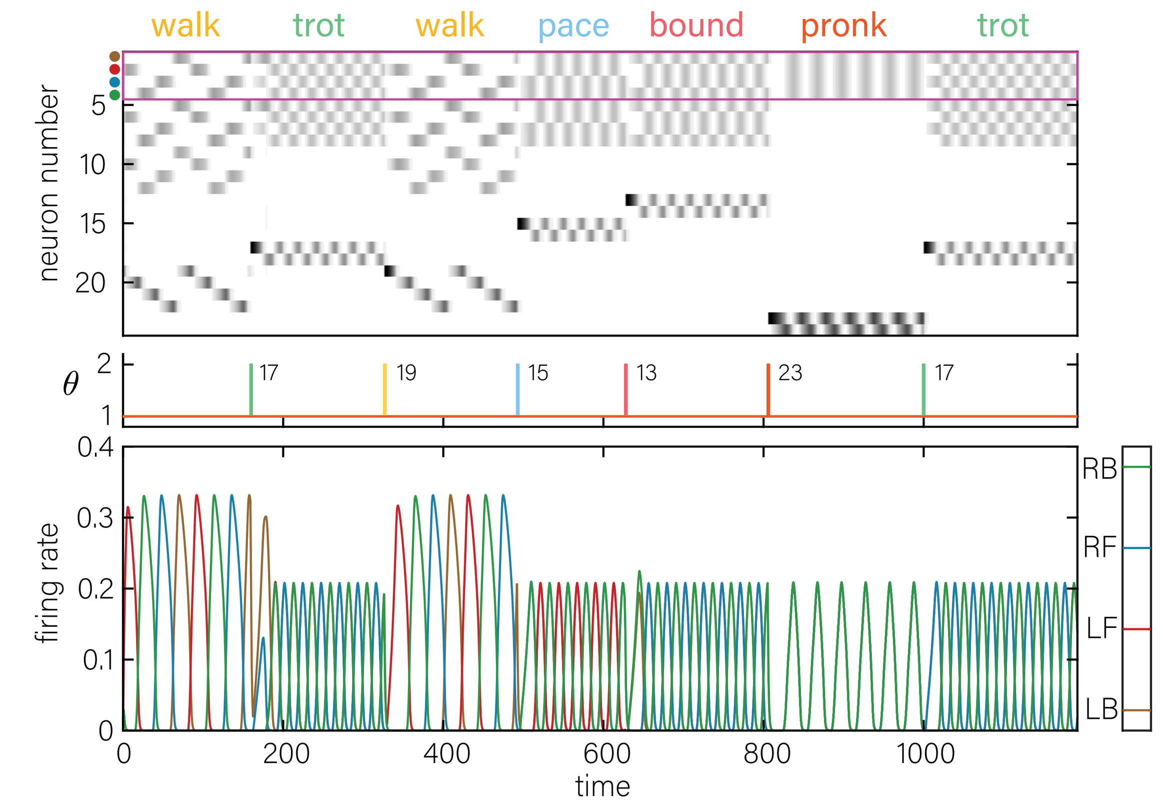
$$\sigma = \text{supp}(\mathbf{x}^*) = \{i \mid x_i^* > 0\} \subseteq \{1, \ldots, n\}$$

There is at most one fixed point per support, so each fixed point can be identified by its support. The collection of all these supports is:

$$\text{FP}(G) = \{\sigma \subseteq [n] \mid \sigma \text{ is the support of a fixed point of a CTLN with graph } G\}$$

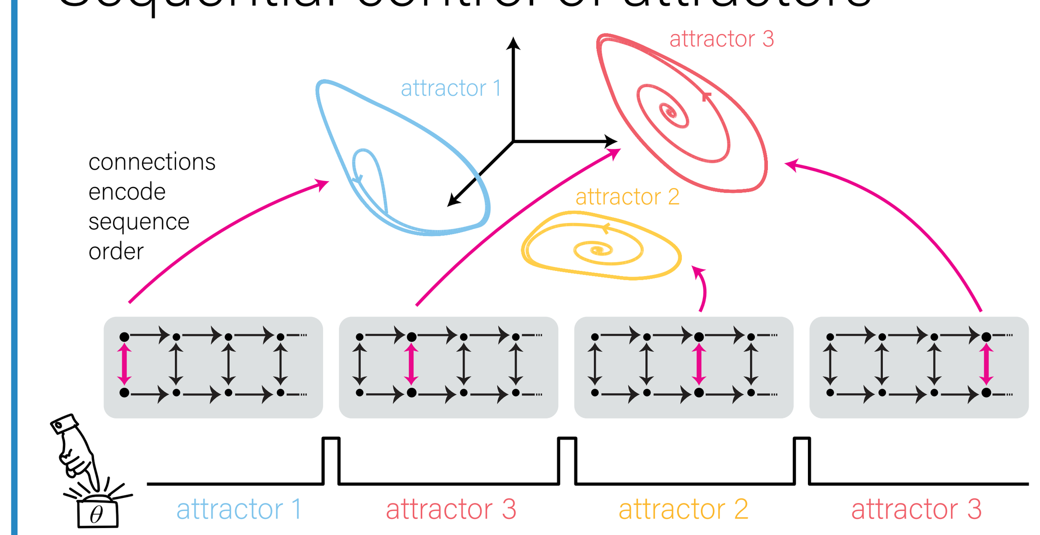
Differences in dynamics are due only to differences in the graph G.

Network can smoothly transition between gaits



Transitioning between gaits within the network is possible **without needing to change parameters**. To change gaits, it suffices to stimulate an auxiliary neuron with a pulse. The network quickly settles into the appropriate dynamic attractor corresponding to the gait of the auxiliary neuron stimulated.

Sequential control of attractors

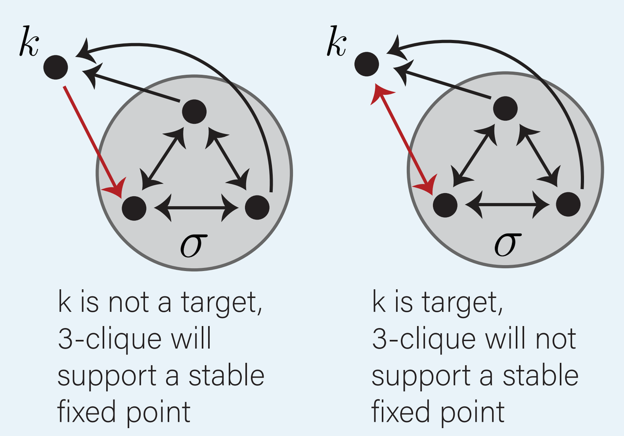


Sequence is encoded separately from attractors. (Dynamic) attractors are preserved for all possible sequences.

Timing of sequence is dissociated from the sequence. A sequence's rhythm can be altered without affecting attractors or changing parameters.

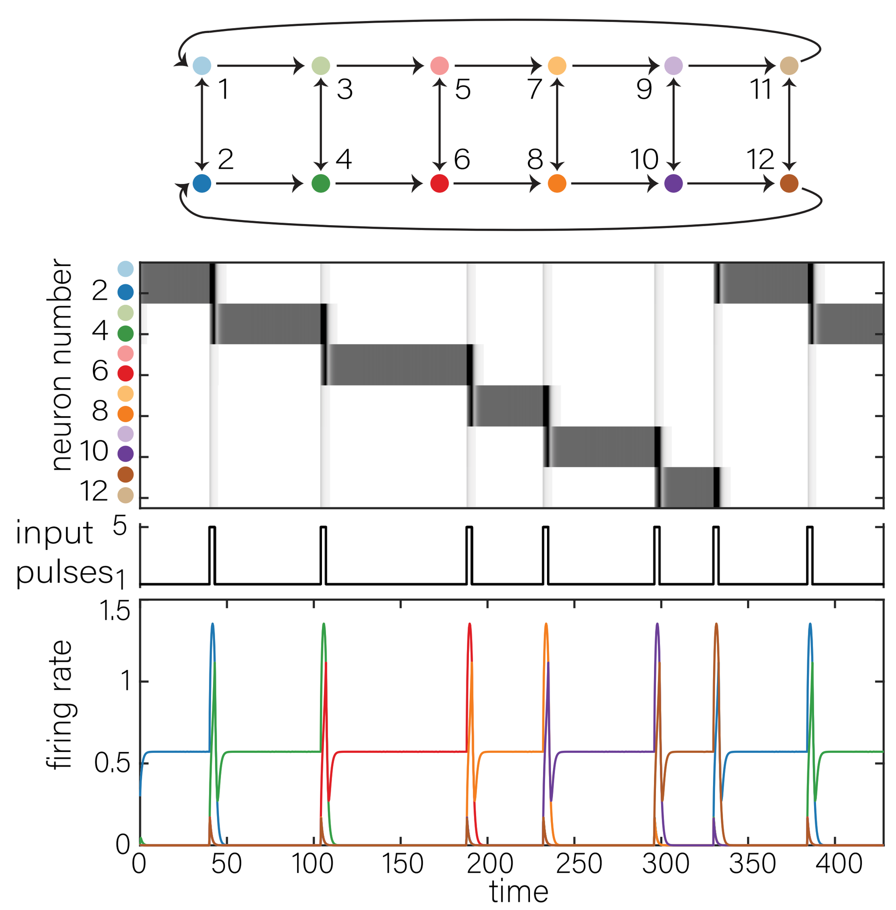
Theorem 1

Theorem: for any graph G, a clique **σ** is the support of a stable fixed point if and only if **σ** is a target-free clique.



Clique: all to all connected graph.

CTLN pulse counter



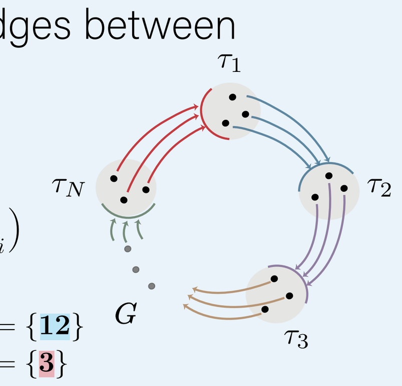
Every 2-clique is target-free and supports a stable fixed point. Pulses are sent to **all** neurons in the network. Activity slides to the next clique (stable fixed point) after reception of the pulse. The network **keeps track of the number of pulse inputs** it has received via the position of the attractor in a linear chain of attractor states (neural integrator model).

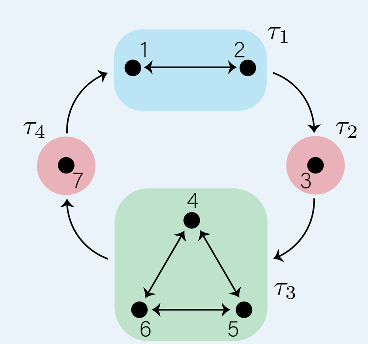
Theorem 2

Cyclic union: edges forward from every node in previous component to every node in next component, and no other edges between components.

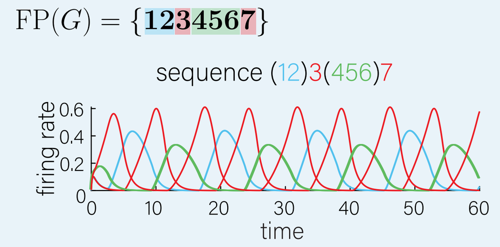
Theorem: for all $i \in [N]$

$$\sigma \in \text{FP}(G) \Leftrightarrow \sigma_i \in \text{FP}(G|_{\tau_i})$$

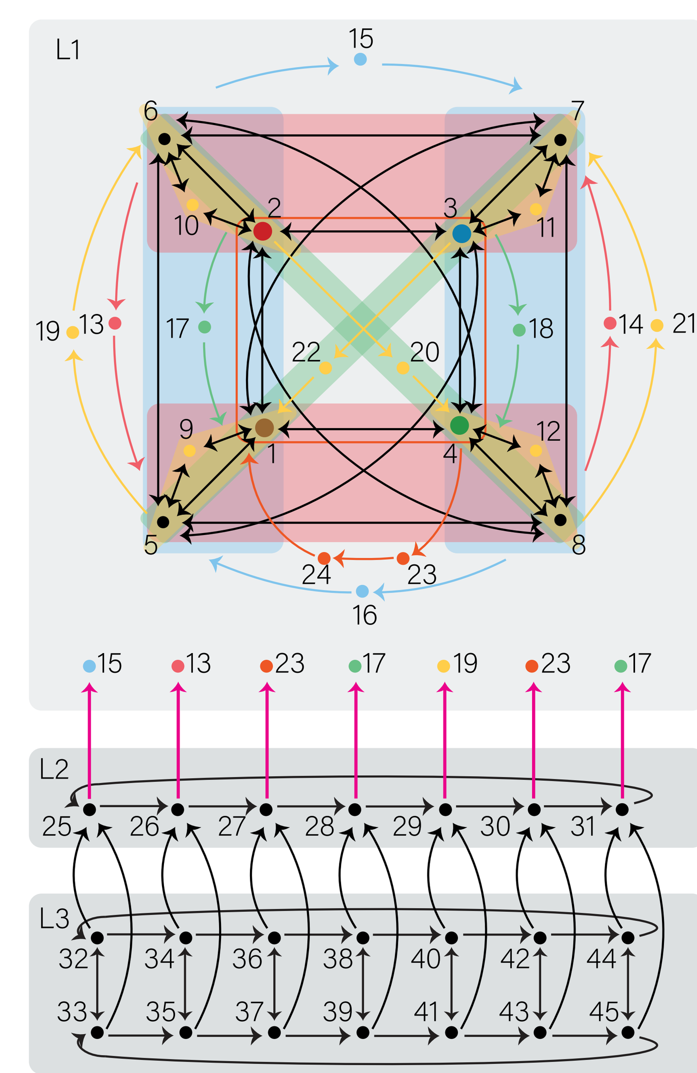


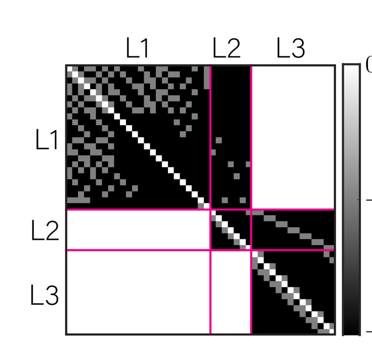


$\text{FP}(G|_{\tau_1}) = \{\mathbf{12}\}$
$\text{FP}(G|_{\tau_2}) = \{\mathbf{3}\}$
$\text{FP}(G|_{\tau_3}) = \{\mathbf{456}\}$
$\text{FP}(G|_{\tau_4}) = \{\mathbf{7}\}$
$\text{FP}(G) = \{\mathbf{1234567}\}$

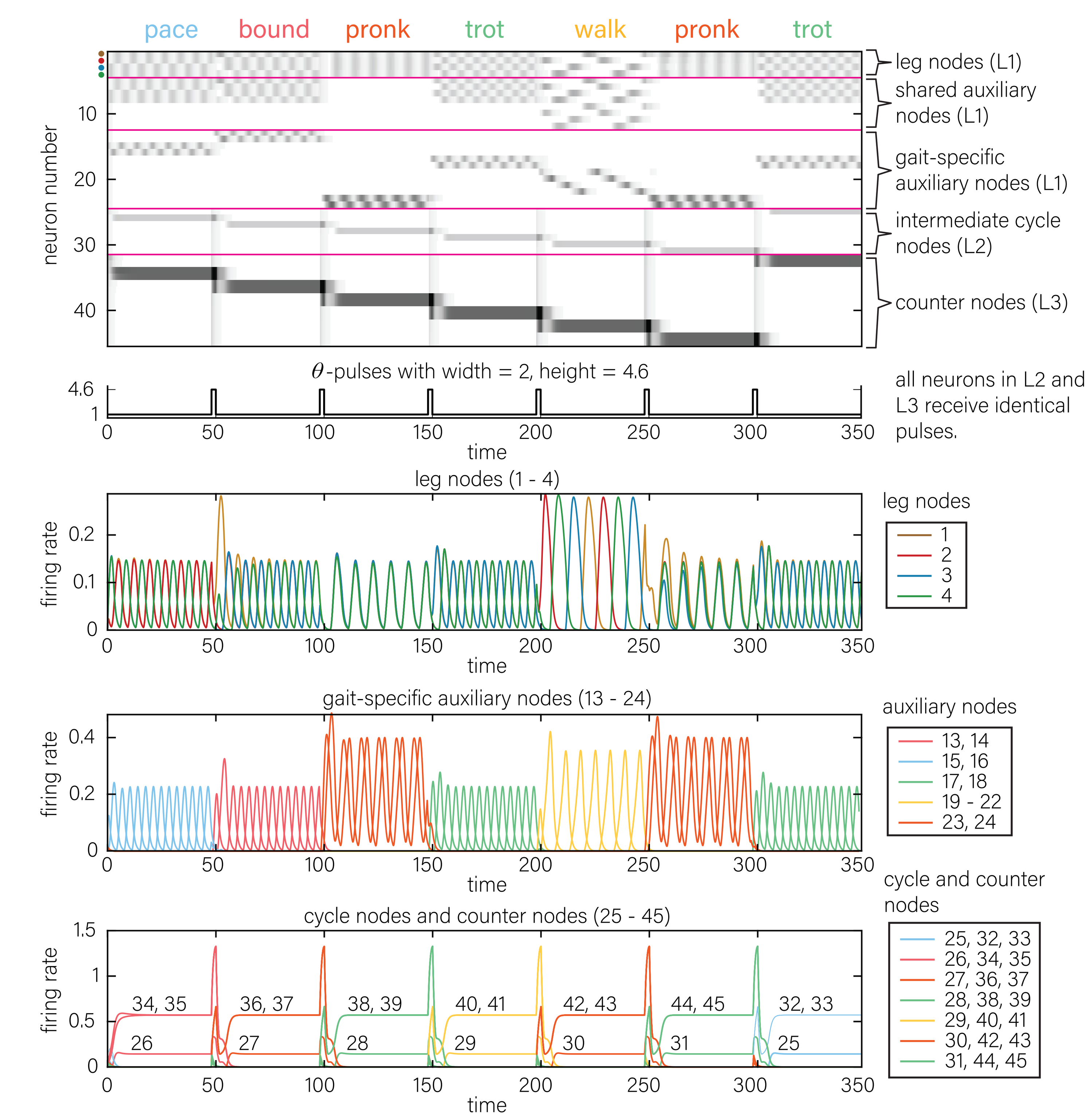


Layered network for sequential control of quadruped gaits



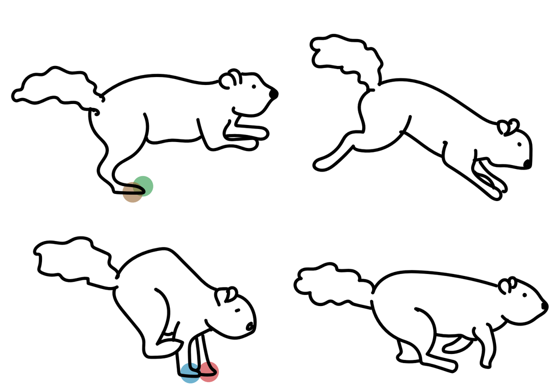


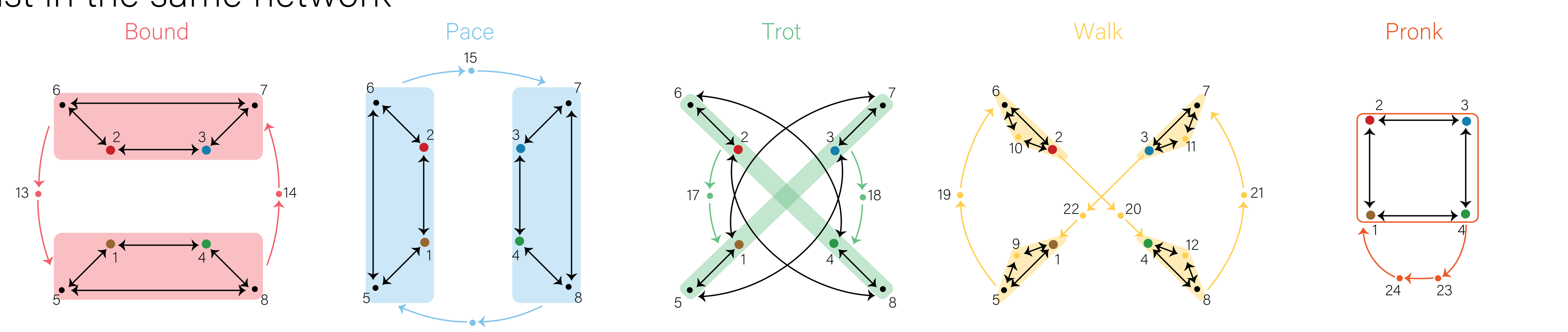
The only inhibition between layers is from L2 to L1 and from L3 to L2.

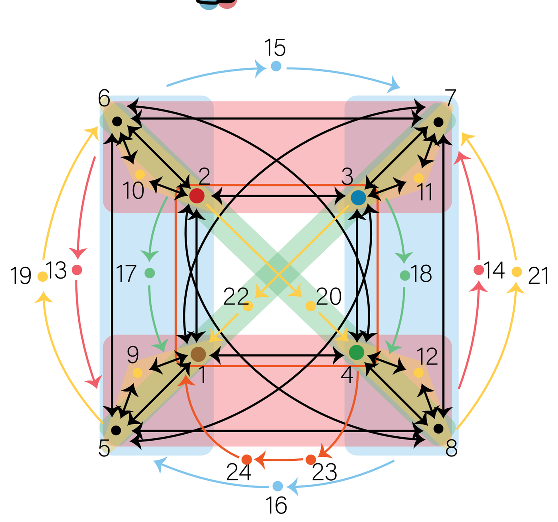


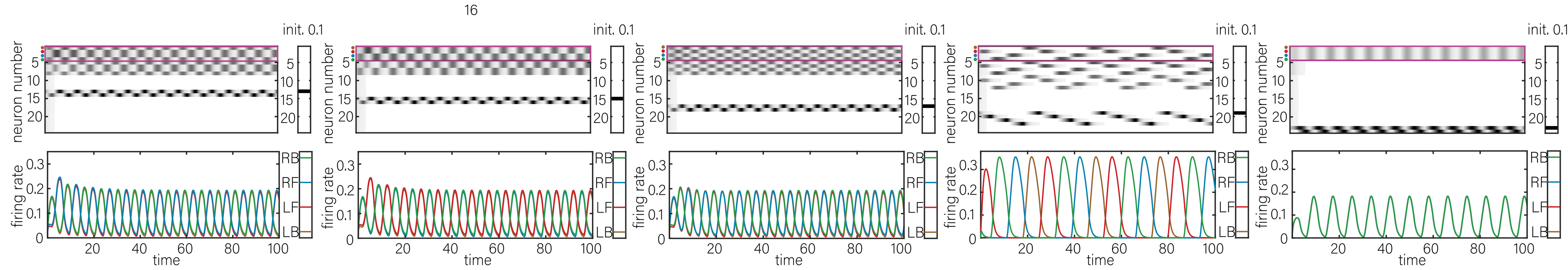
Gaits will be accessed in the order specified in the connections from L2 to L1 (auxiliary nodes corresponding to different gaits redrawn at the bottom of L1 for clarity). Only neurons in L2 and L3 receive pulses, and so the sequence is stored within the network. Each pulse moves L2 and L3 one step to the right, activating the next gait down the sequence. Cycle nodes and counter nodes are one step out of synch with the gait sequence. **Any sequence can be encoded** between layers L2 and L3 and layer L1.

Five gaits can coexist in the same network









Each gait is modeled as a **cyclic union**. Graphs can be "glued" together to produce a single network where all five gaits **coexist as distinct limit cycle attractors** in the network and can be individually accessed **without needing to change parameters**. Instead, different gaits are obtained by changes in initial conditions, or by stimulation of a specific neuron uniquely involved in the gait.

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