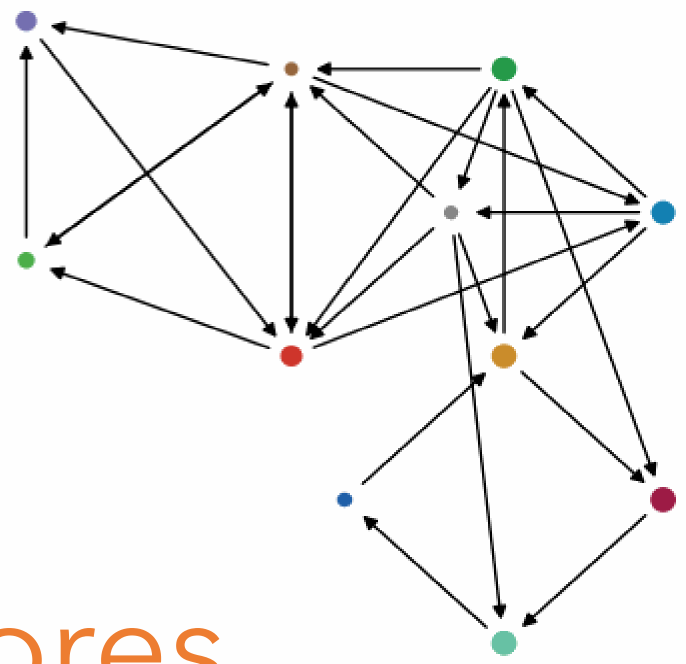
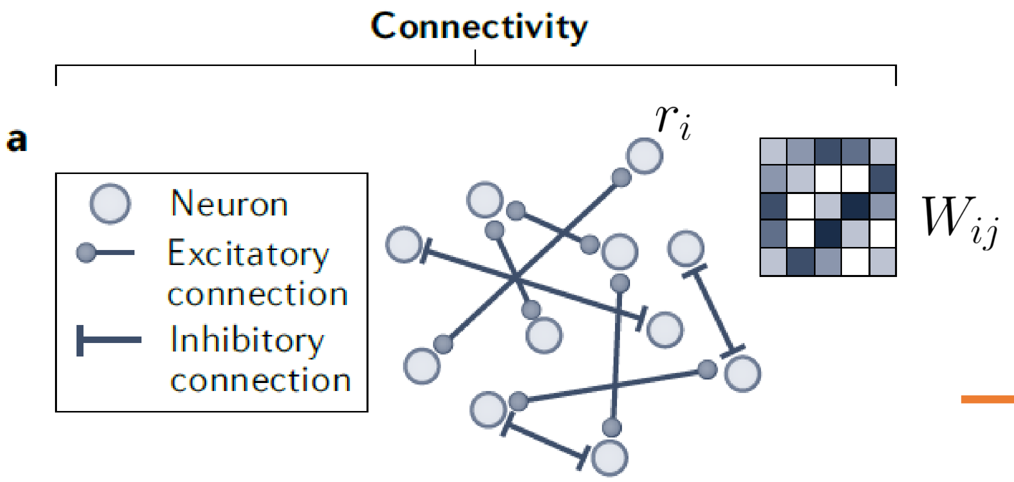


Control secuencial de atractores dinámicos usando redes tipo CTLN

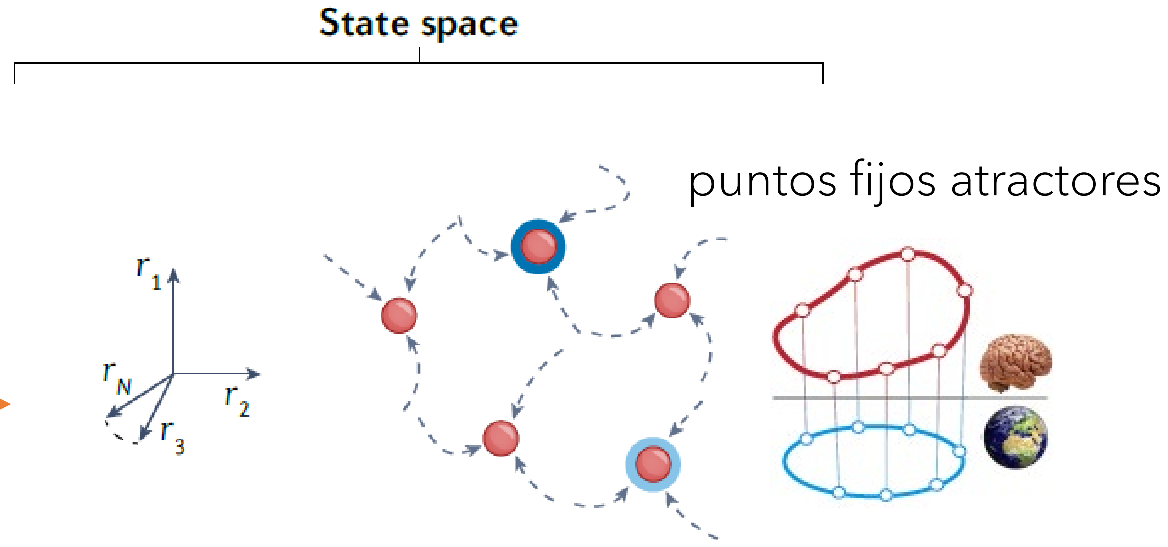
Juliana Londoño Álvarez
(with Carina Curto and Katie Morrison)
Pennsylvania State University
June 2023

Los atractores como representaciones de procesos cognitivos



$$\frac{dr_i}{dt} = -r_i + \varphi\left(\sum_{j=1}^{n} W_{ij} r_j + b_i\right)$$

Red recurrente,
dinámica

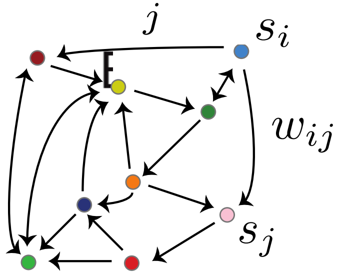


Red neuronal de atractores

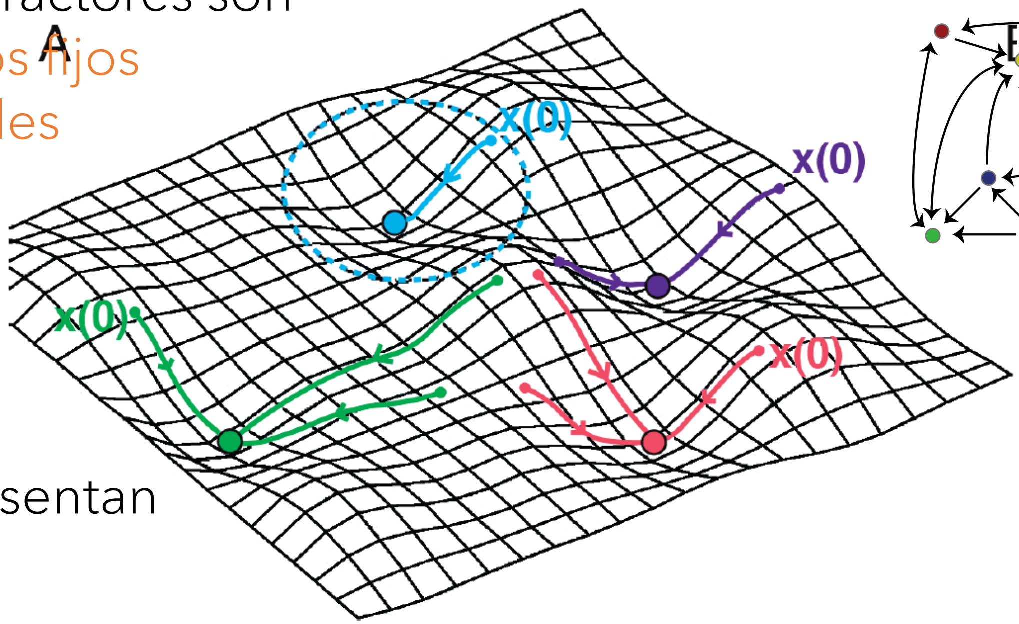
Khona, M., Fiete, I.R. Attractor and integrator networks in the brain. Nat Rev Neurosci 23, 744–766 (2022).

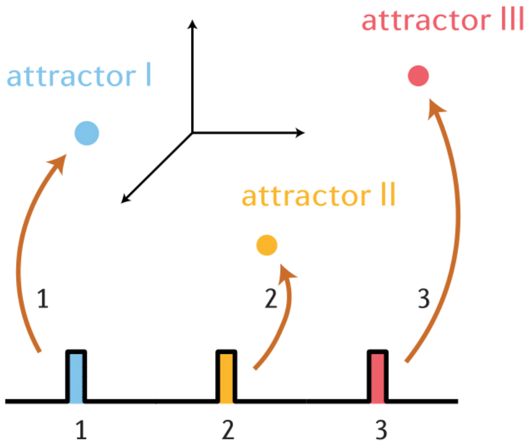
Redes Hopfield (1982)

$$s_i(t+1) = \mathrm{sgn}\Big(\sum_j w_{ij} s_j(t) - \theta_i\Big)$$



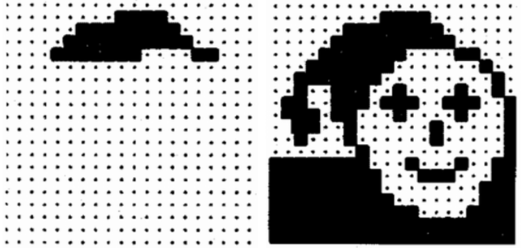
Los atractores son puntos fijos estables





Atractores representan memorias

Las secuencia están codificadas externamente



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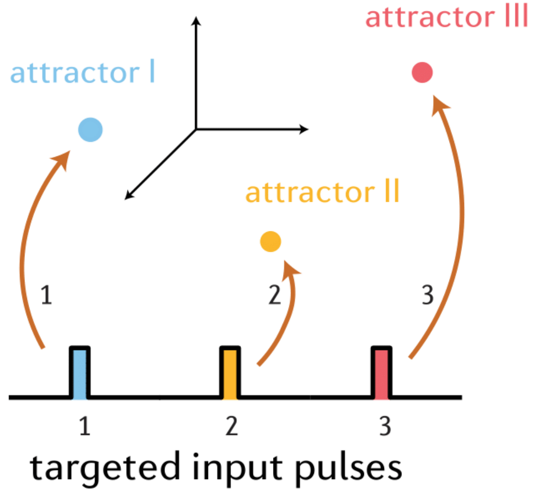
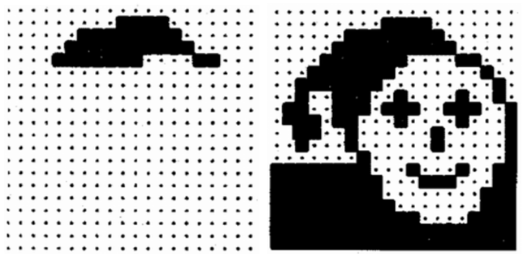
Los inputs son específicos para cada atractor

Roadmap

A classical Hopfield-like paradigm





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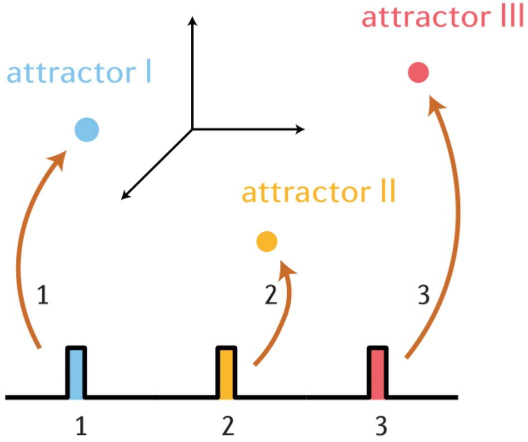
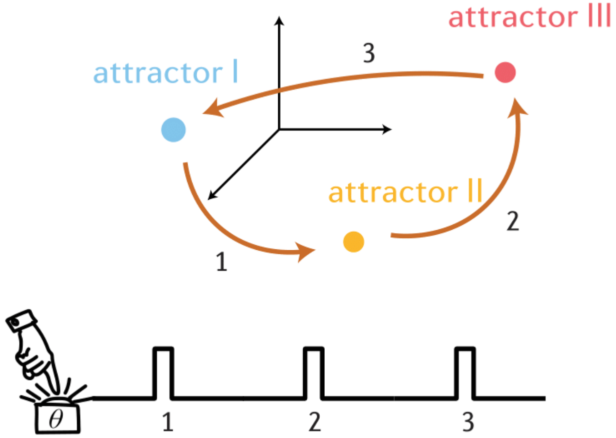
Roadmap

A classical Hopfield-like paradigm

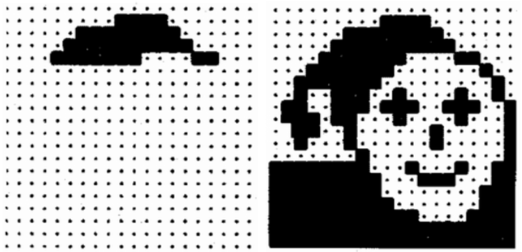
B sequential activation of static attractors



targeted input pulses



identical input pulses



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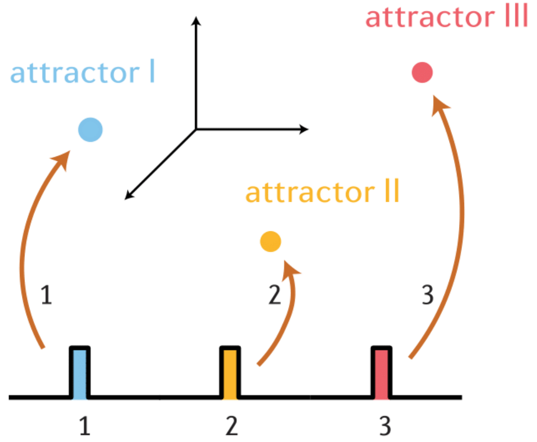
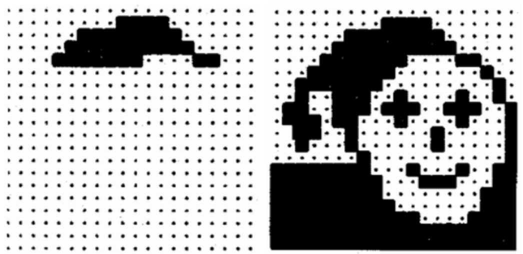
Roadmap

A classical Hopfield-like paradigm



targeted input pulses



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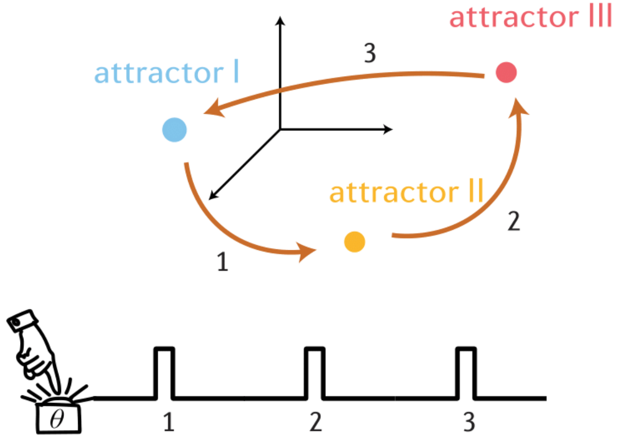
B sequential activation of static attractors



identical input pulses

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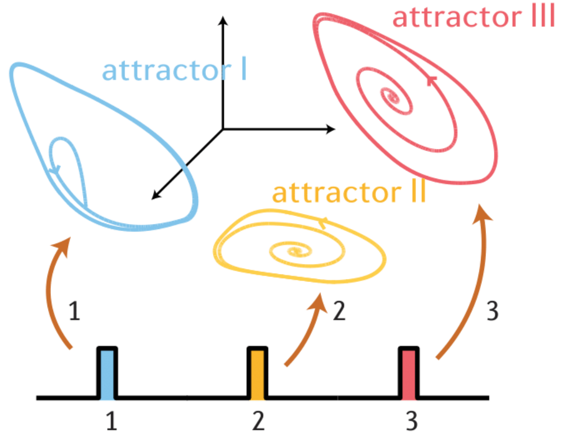
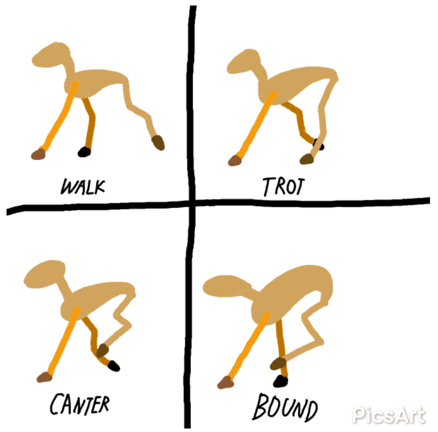
C coexistent dynamic attractors



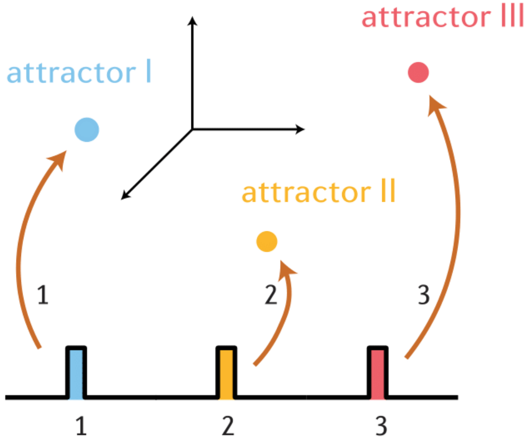
targeted input pulses



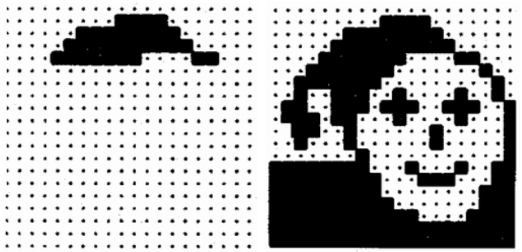
Reddit?

Roadmap

A classical Hopfield-like paradigm

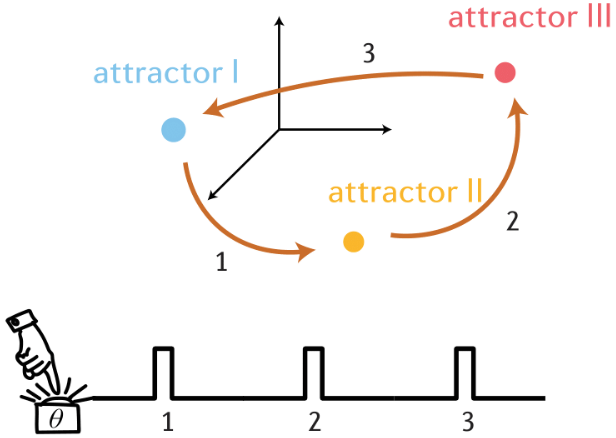


targeted input pulses



Holk Cruse, Neural Networks as Cybernetic Systems

B sequential activation of static attractors



identical input pulses

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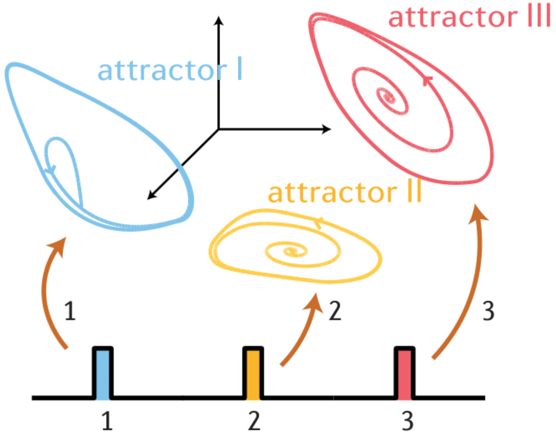
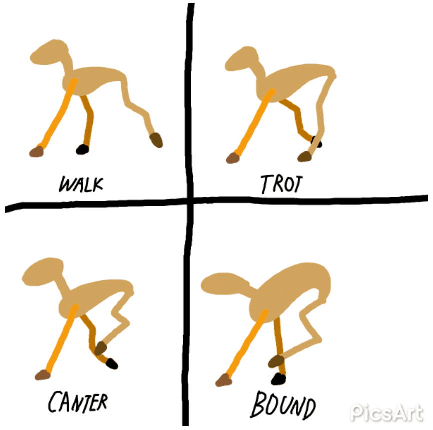
C coexistent dynamic attractors

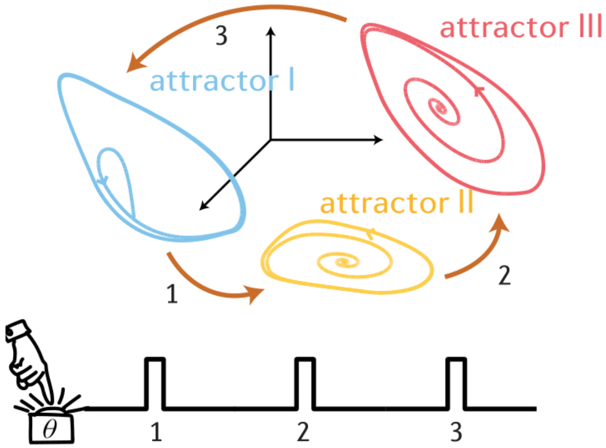


targeted input pulses

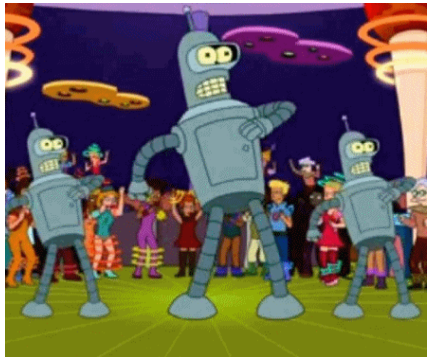


Reddit?

D sequential activation of dynamic attractors

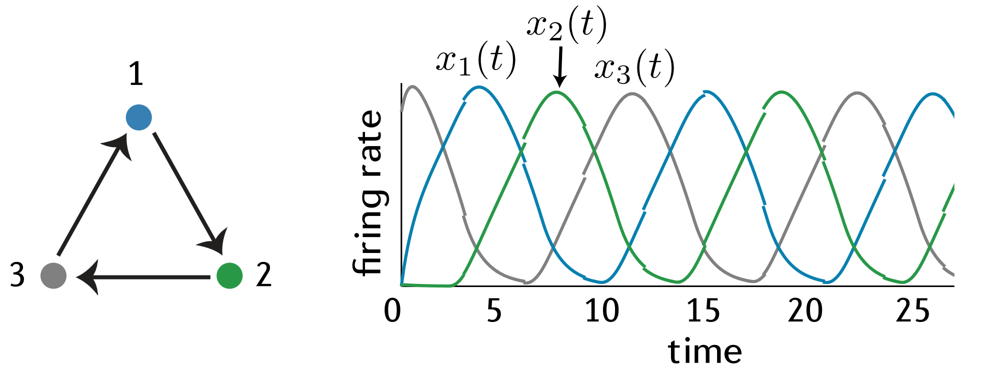


identical input pulses



0. Marco teórico

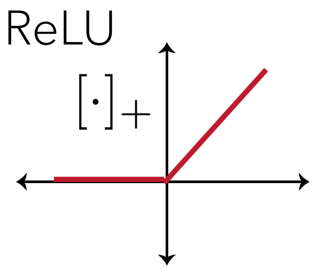
$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij} x_j + b_i \right]_+$$

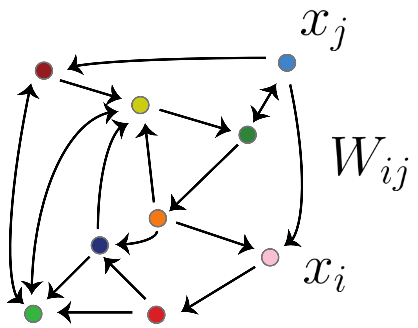


Combinatorial threshold-linear networks (CTLNs)

Redes neuronales recurrentes con no linearidad ReLU, cuya matriz de conectividad está prescrita por un grafo dirigido

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+, \quad i = 1, \ldots, n,$$





$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1 + \varepsilon & \text{if } j \to i \text{ in } G, \\ -1 - \delta & \text{if } j \not\to i \text{ in } G. \end{cases}$$

j weakly inhibits i

j strongly inhibits i

$$b_i = \theta$$ Constant input

$$\epsilon = 0.25, \delta = 0.5, \theta = 1 \qquad \theta, \delta > 0 \qquad 0 < \varepsilon < \frac{\delta}{\delta + 1}$$

Todas las conexiones son inhibitorias

Las (C)TLNs son EDOs lineales por partes

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

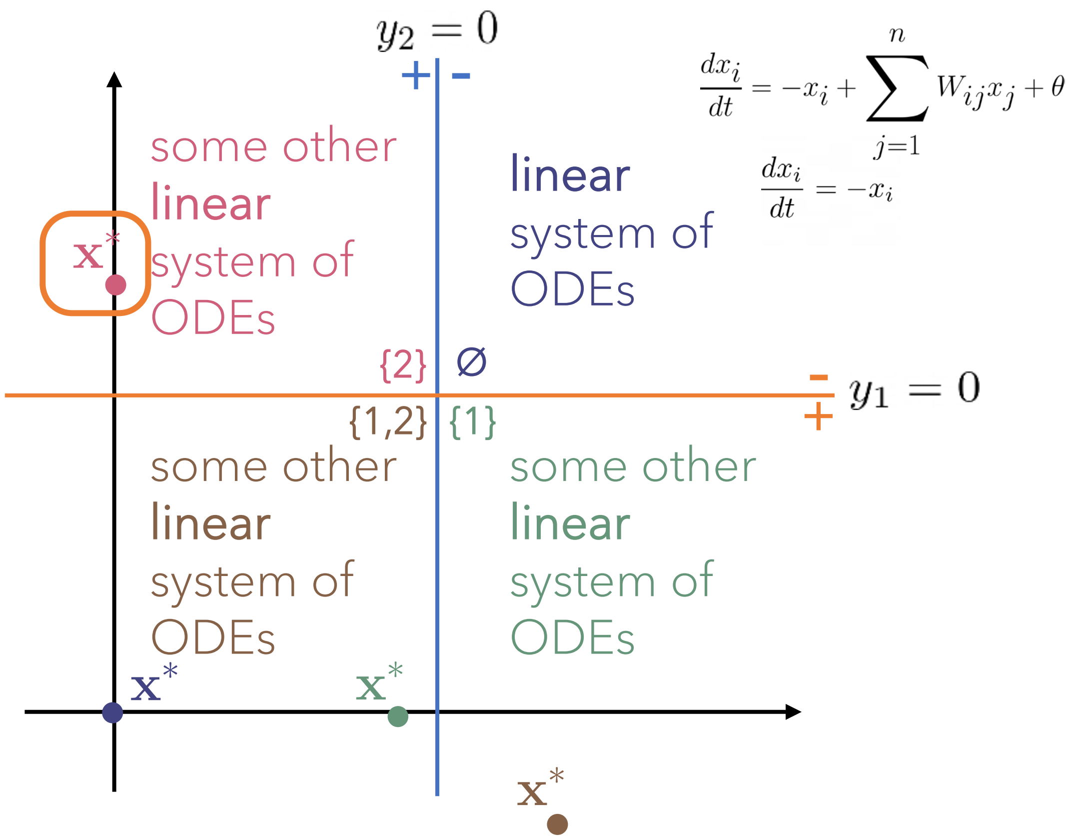
En un punto fijo: $x_i = \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+ = [y_i]_+$

Podemos etiquetar todos los puntos fijos de la red por su soporte:

$$\sigma = \mathrm{supp}(\mathbf{x}^*) = \{i \,|\, x_i > 0\} \subseteq \{1, \ldots, n\}$$

y denotamos esa colección por

$$\mathrm{FP}(G) = \{\sigma \subseteq [n] \mid \sigma \text{ is the support of a fixed point of the associated CTLN}\}$$



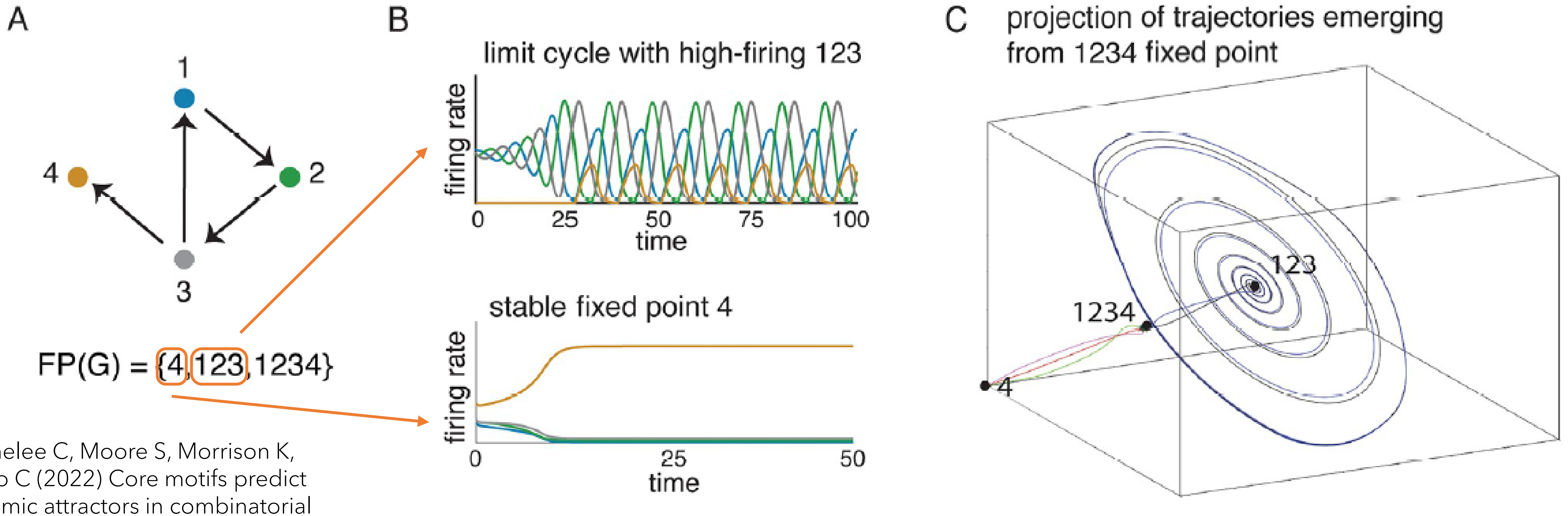
Es fácil obtener el punto fijo a partir de su soporte:

$$x_k^* = 0 \text{ for all } k \notin \sigma$$
$$\boldsymbol{x}_\sigma^* = (I - W_\sigma)^{-1}\boldsymbol{\theta}_\sigma$$

Soportes minimales

FP$(G) = \{\sigma \subseteq [n] | \sigma$ is the support of a fixed point of the CTLN with graph $G\}$

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$



Parmelee C, Moore S, Morrison K, Curto C (2022) Core motifs predict dynamic attractors in combinatorial threshold-linear networks. PLOS ONE 17(3): e0264456.

Heurística: los atractores estáticos corresponden a cliques (soportes mínimos) y los atractores dinámicos corresponden a soportes mínimos que no son cliques.

Dado un conjunto de atractores, ¿qué arquitectura de red podría generarlos?

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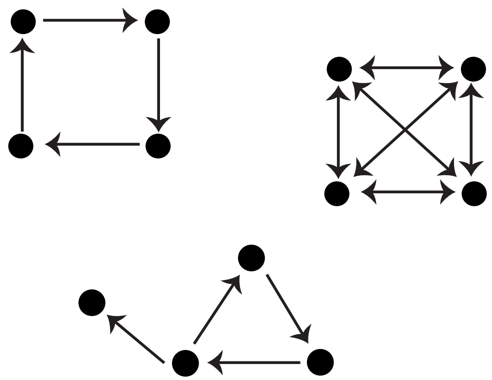
$$\mathrm{FP}(G, \varepsilon, \delta)$$

Soportes minimales

Dado un conjunto de atractores, ¿qué arquitectura de red podría generarlos?

$$\mathrm{FP}(G, \varepsilon, \delta)$$

Soportes minimales



Arquitectura del grafo

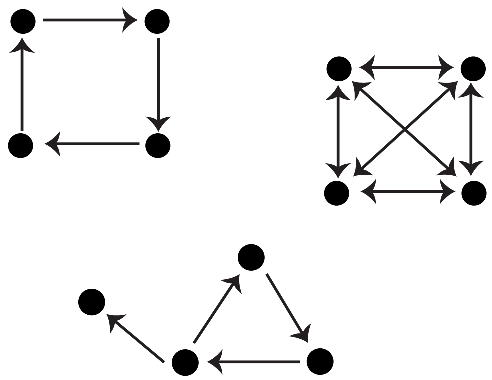
Dado un conjunto de atractores, ¿qué arquitectura de red podría generarlos?

$$\mathrm{FP}(G, \varepsilon, \delta)$$

Soportes minimales

Reglas gráficas para CTLNs

Curto, C., & Morrison, K. (2023). Graph Rules for Recurrent Neural Network Dynamics. Notices of the AMS.

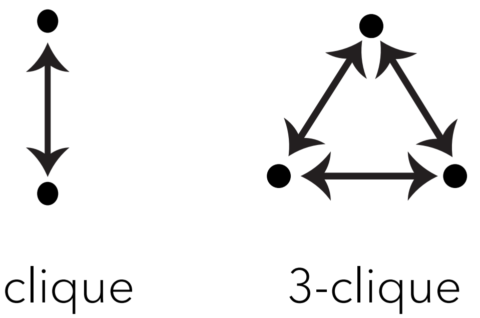


Arquitectura del grafo

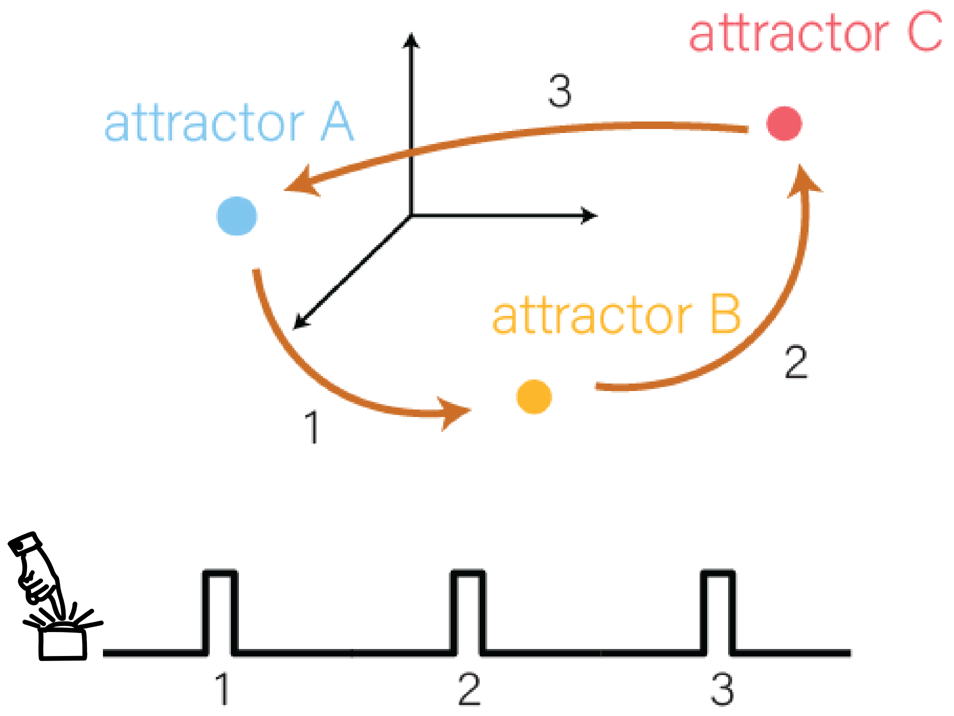
Reglas gráficas para CTLNs

Rule name	$G\|_\sigma$ structure	graph rule
Rule 1	independent set	$\sigma \in \mathrm{FP}(G\|_\sigma)$, and $\sigma \in \mathrm{FP}(G) \Leftrightarrow \sigma$ is a union of sinks
Rule 2	clique	$\sigma \in \mathrm{FP}(G\|_\sigma)$, and $\sigma \in \mathrm{FP}(G) \Leftrightarrow \sigma$ is target-free
Rule 3	cycle	$\sigma \in \mathrm{FP}(G\|_\sigma)$, and $\sigma \in \mathrm{FP}(G) \Leftrightarrow$ each $k \notin \sigma$ receives at most one edge $i \to k$ with $i \in \sigma$
Rule 4(i)	$\exists$ a source $j \in \sigma$	$\sigma \notin \mathrm{FP}(G)$ if $j \to k$ for some $k \in [n]$
Rule 4(ii)	$\exists$ a source $j \notin \sigma$	$\sigma \in \mathrm{FP}(G\|_{\sigma \cup j}) \Leftrightarrow \sigma \in \mathrm{FP}(G\|_\sigma)$
Rule 5(i)	$\exists$ a target $k \in \sigma$	$\sigma \notin \mathrm{FP}(G\|_\sigma)$ and $\sigma \notin \mathrm{FP}(G)$ if $k \not\to j$ for some $j \in \sigma$
Rule 5(ii)	$\exists$ a target $k \notin \sigma$	$\sigma \notin \mathrm{FP}(G\|_{\sigma \cup k})$ and $\sigma \notin \mathrm{FP}(G)$
Rule 6	$\exists$ a sink $s \notin \sigma$	$\sigma \cup \{s\} \in \mathrm{FP}(G) \Leftrightarrow \sigma \in \mathrm{FP}(G)$
Rule 7	DAG	$\mathrm{FP}(G) = \{\cup s_i \mid s_i$ is a sink in $G\}$
Rule 8	arbitrary	$\|\mathrm{FP}(G)\|$ is odd

σ soporta un punto fijo estable!



Carina Curto, & Katherine Morrison. (2023). Graph rules for recurrent neural network dynamics: extended version.

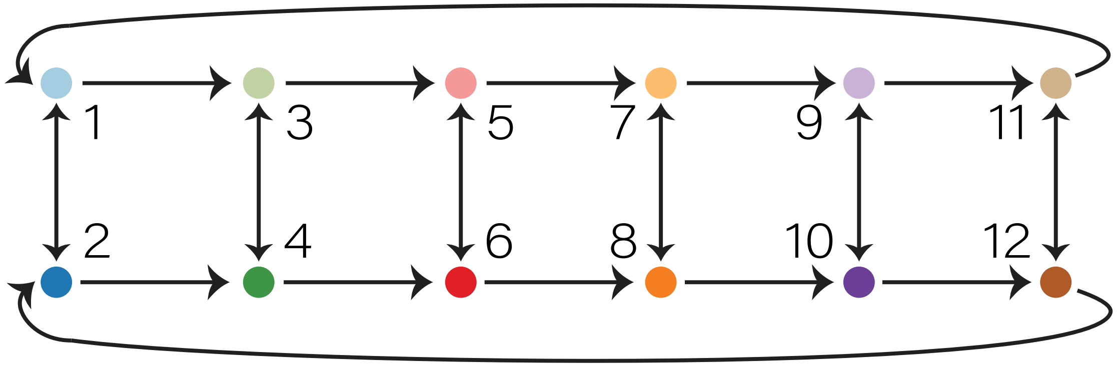


Q1. ¿Puede una secuencia de activaciones ser codificada, de manera flexible, de modo que se active con inputs idénticos?

Ejemplo: contador CTLN

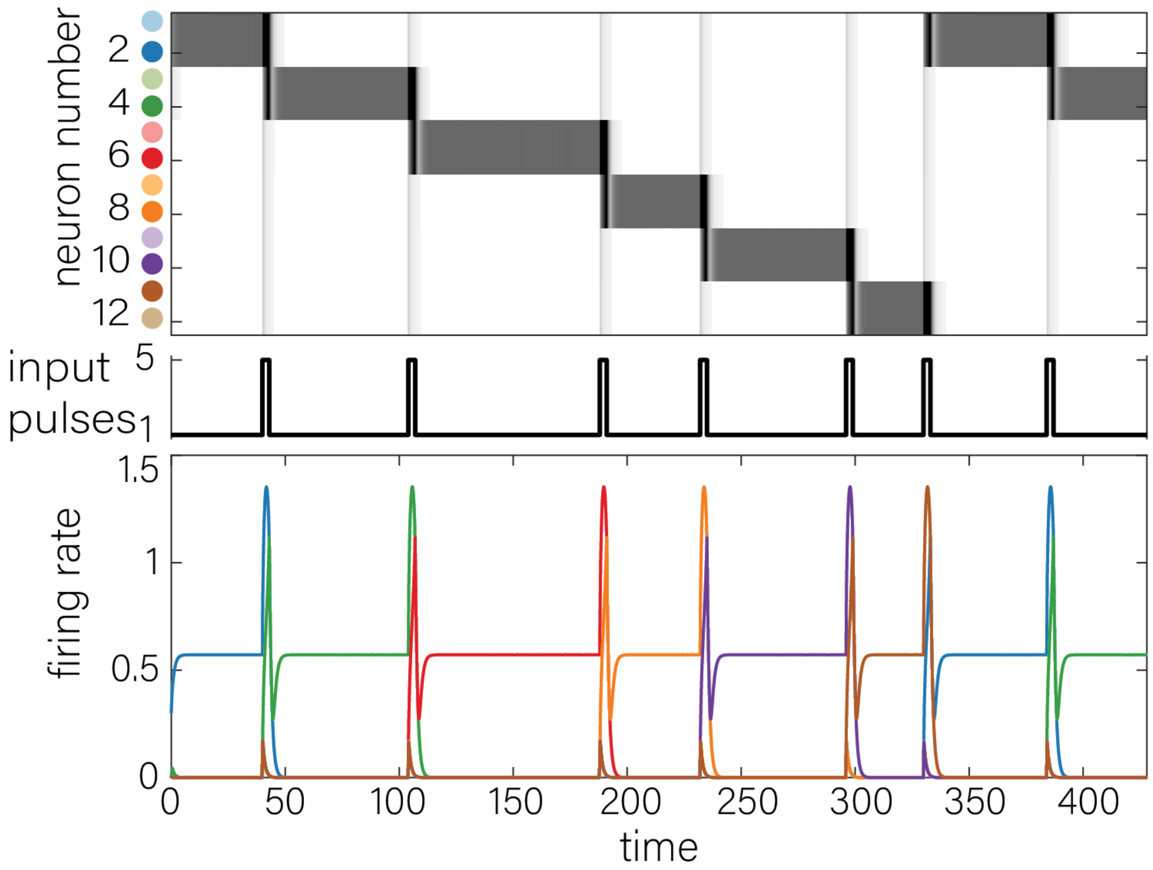
$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

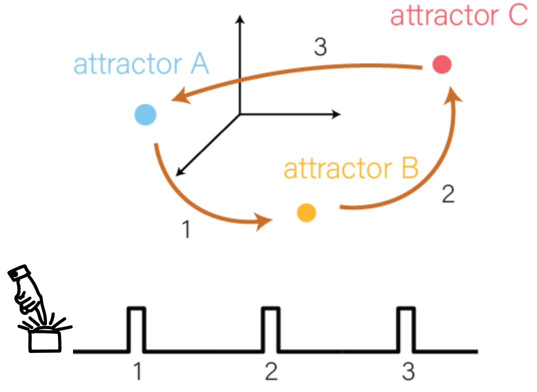
Cadena de cliques libres de blancos, cada 2-clique es un punto fijo estable

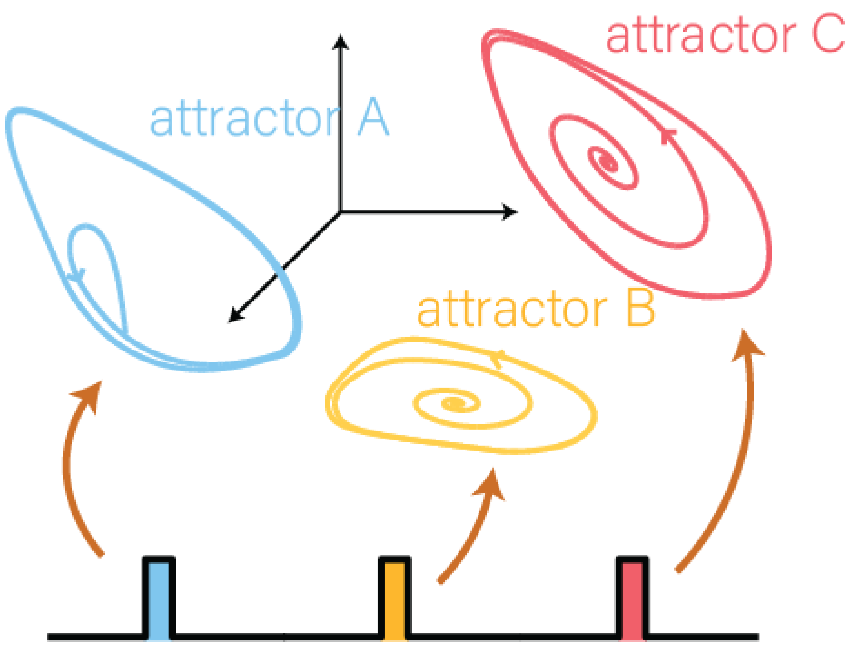


$$|\mathrm{FP}(G)| = 141$$

$$\text{minimal supports} = \{\{1,2\},\{3,4\},\{5,6\},\{7,8\},\{9,10\},$$
$$\{11,12\},\{1,3,5,7,9,11\},\{2,4,6,8,10,12\}\}$$



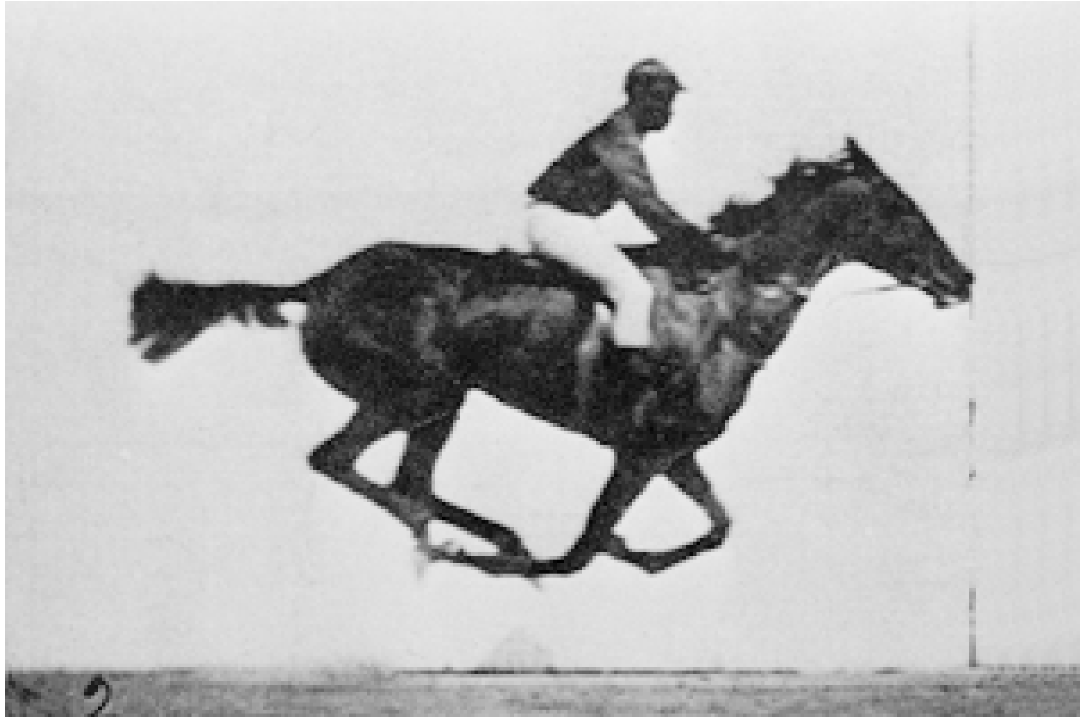




Q1. ¿Pueden múltiples atractores dinámicos ser codificados y activados mediante inputs específicos, en una sola red?

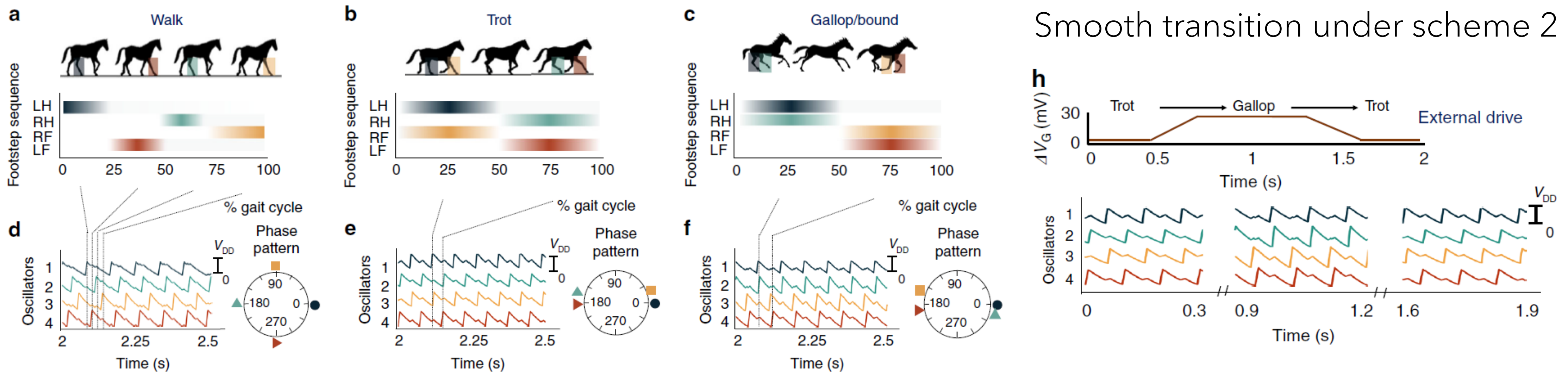
Model problem: quadruped locomotion

Cuando un caballo está trotando o galopando, ¿tienen los cuatro pies del caballo al mismo tiempo fuera del suelo?



Eadweard Muybridge, 1878.

Modelos de locomoción, ahora (2019).

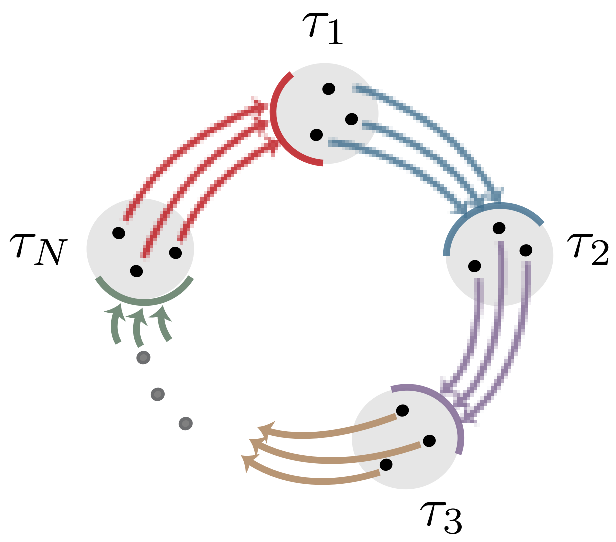


Dutta, S., Parihar, A., Khanna, A. *et al.* Programmable coupled oscillators for synchronized locomotion. *Nat Commun* 10, 3299 (2019).

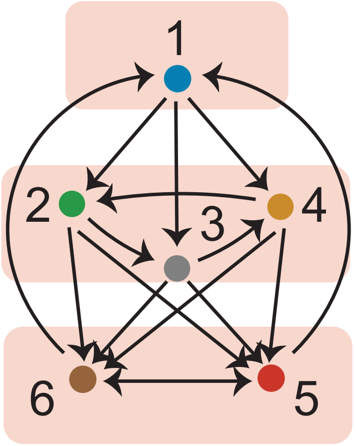
- ¿Cómo coexisten los andares en una sola red?
- ¿Podemos hacer la transición entre diferentes andares?
- ¿Cómo podemos reducir el número de parámetros?

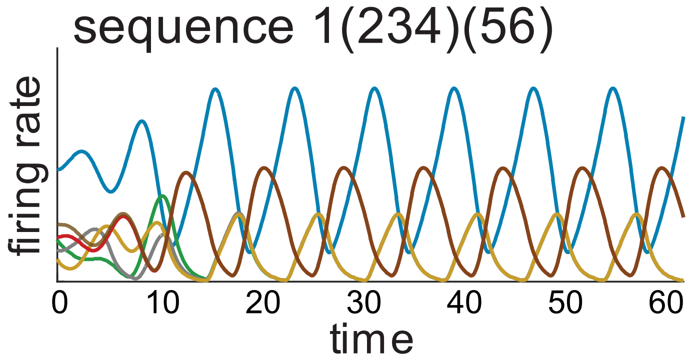
Las uniones cíclicas producen activaciones rítmicas.

Unión cíclica: aristas que van desde cada nodo en la componente anterior hacia cada nodo en la siguiente componente, y ninguna otra arista entre componentes.



$$\mathrm{FP}\left(G|_{\{1\}}\right) = \{1\}$$
$$\mathrm{FP}\left(G|_{\{2,3,4\}}\right) = \{2,3,4\}$$
$$\mathrm{FP}\left(G|_{\{5,6\}}\right) = \{5,6\}$$





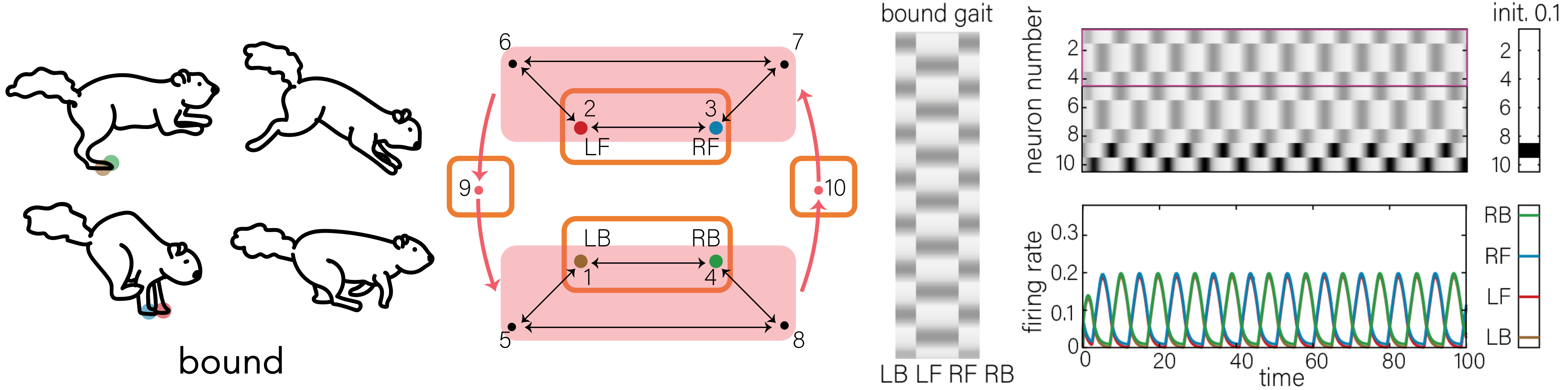
$$\mathrm{FP}(G) = \{1,2,3,4,5,6\}$$

Theorem: for all $i \in [N]$ $\quad \sigma \in \mathrm{FP}(G) \Leftrightarrow \sigma \cap \tau_i \in \mathrm{FP}(G|_{\tau_i})$

Parmelee, C., **Londono Alvarez, J.,** Curto, C., & Morrison, K. (2022). Sequential attractors in combinatorial threshold-linear networks. *SIAM Journal on Applied Dynamical Systems*, 21(2), 1597-1630.

Ejemplo de generador central de patrones: un solo *andar*

Las patas delanteras están sincronizadas, las patas traseras están sincronizadas, separadas por medio período.

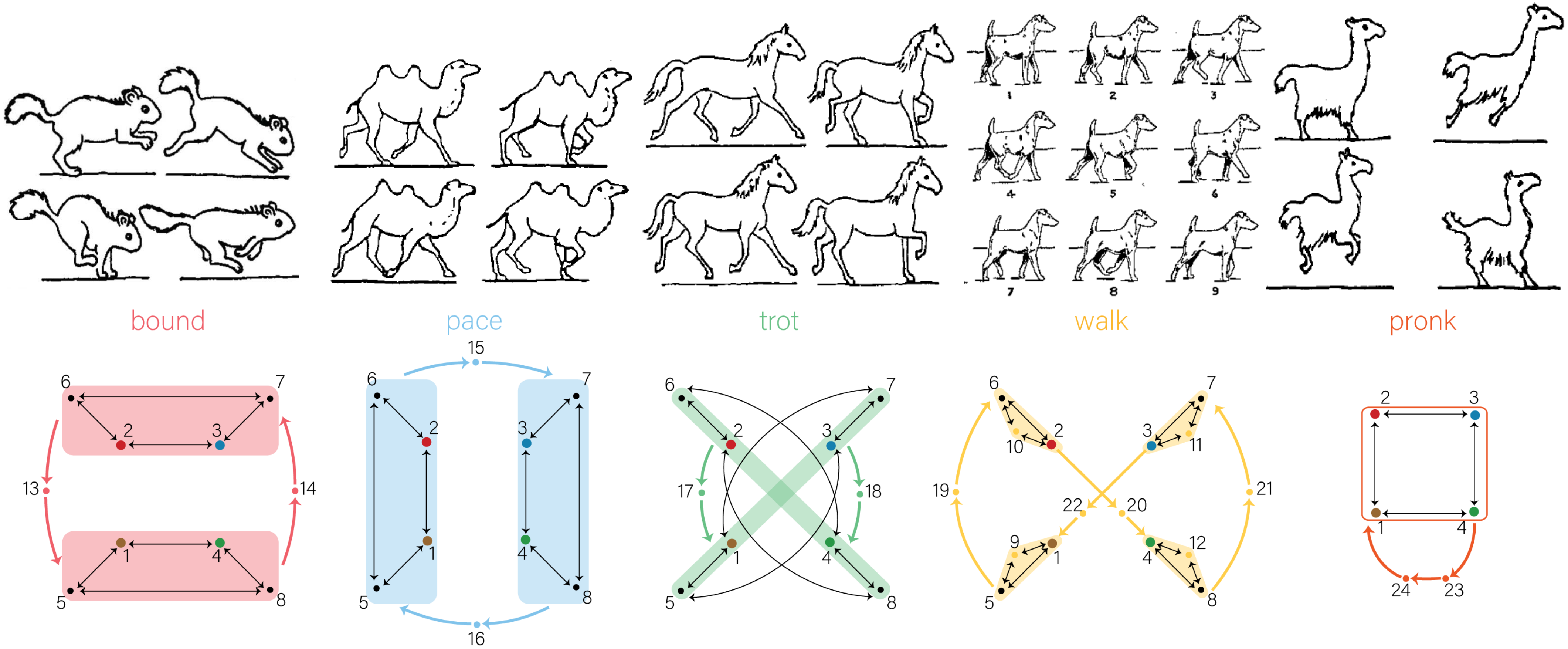


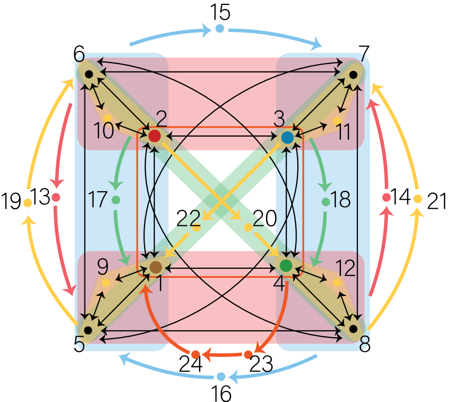
$$|\mathrm{FP}(G)| = 81$$

$$\begin{aligned}\text{minimal supports } = \{&\boxed{\{1,2,3,4,9,10\}}, \{1,2,3,5,9,10\}, \{1,2,4,6,9,10\}, \{1,2,5,6,9,10\}, \{1,3,4,7,9,10\},\\ &\{1,3,5,7,9,10\}, \{1,4,6,7,9,10\}, \{1,5,6,7,9,10\}, \{2,3,4,8,9,10\}, \{2,3,5,8,9,10\}, \{2,4,6,8,9,10\},\\ &\{2,5,6,8,9,10\}, \{3,4,7,8,9,10\}, \{3,5,7,8,9,10\}, \{4,6,7,8,9,10\}, \{5,6,7,8,9,10\}\}\end{aligned}$$

Red con 5 *andares*

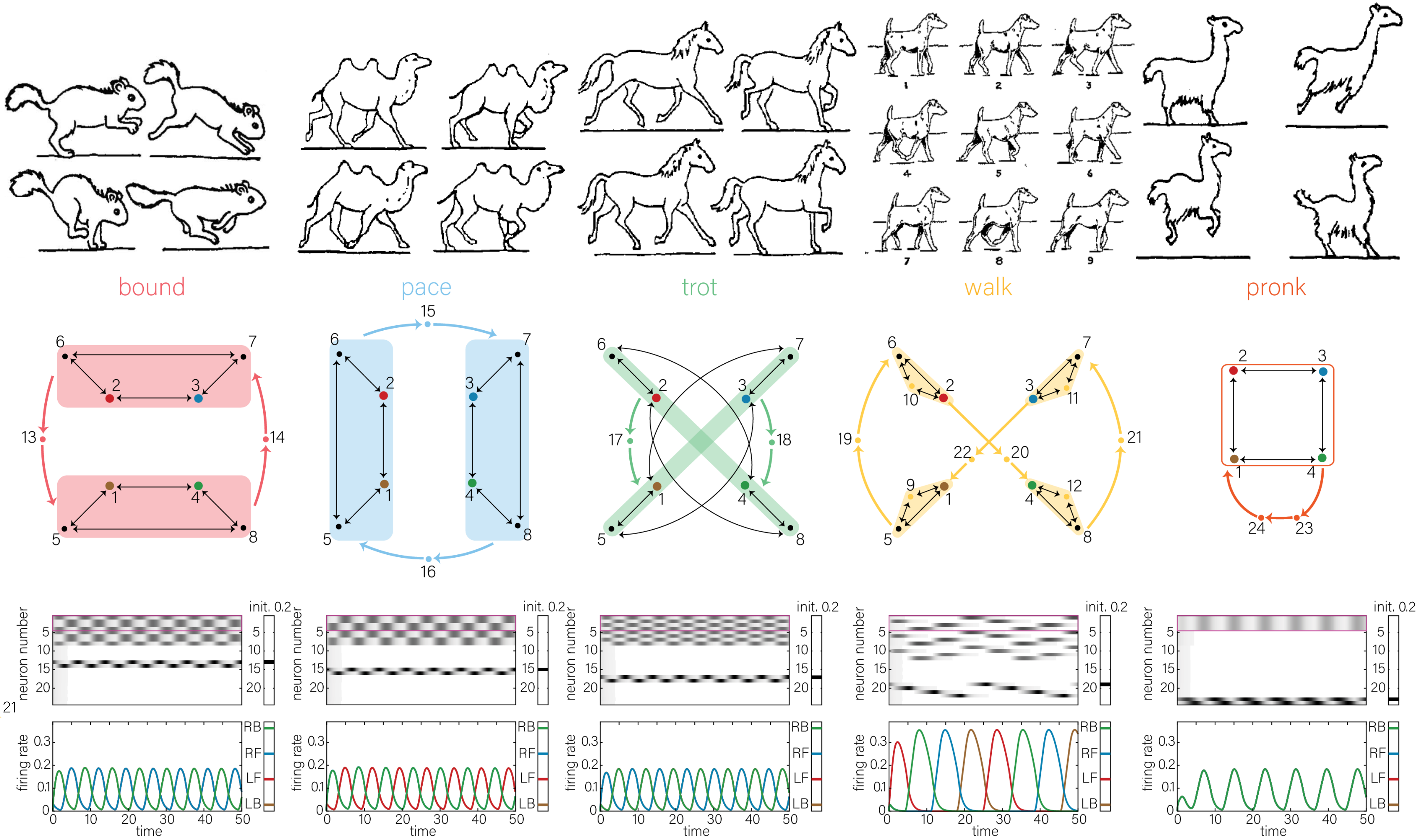
Dibujos: van der Weele, J. P., & Banning, E. J. (2001). *American Journal of Physics*.



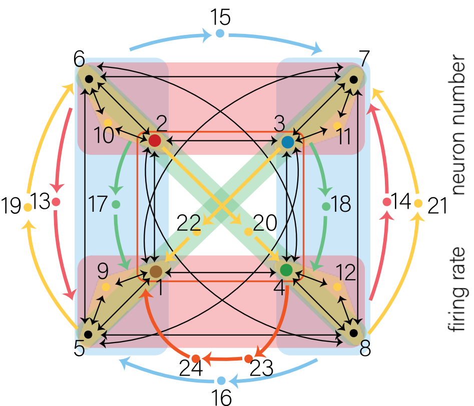


Red con 5 *andares*

Dibujos: van der Weele, J. P., & Banning, E. J. (2001). *American Journal of Physics.*



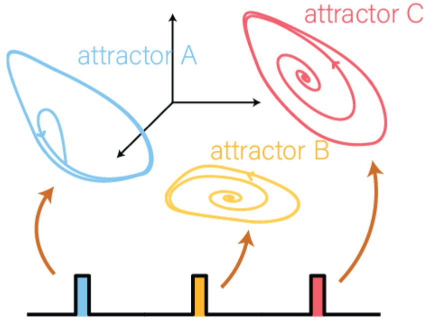
Todos los modos de andar son accesibles mediante cambios en las condiciones iniciales.

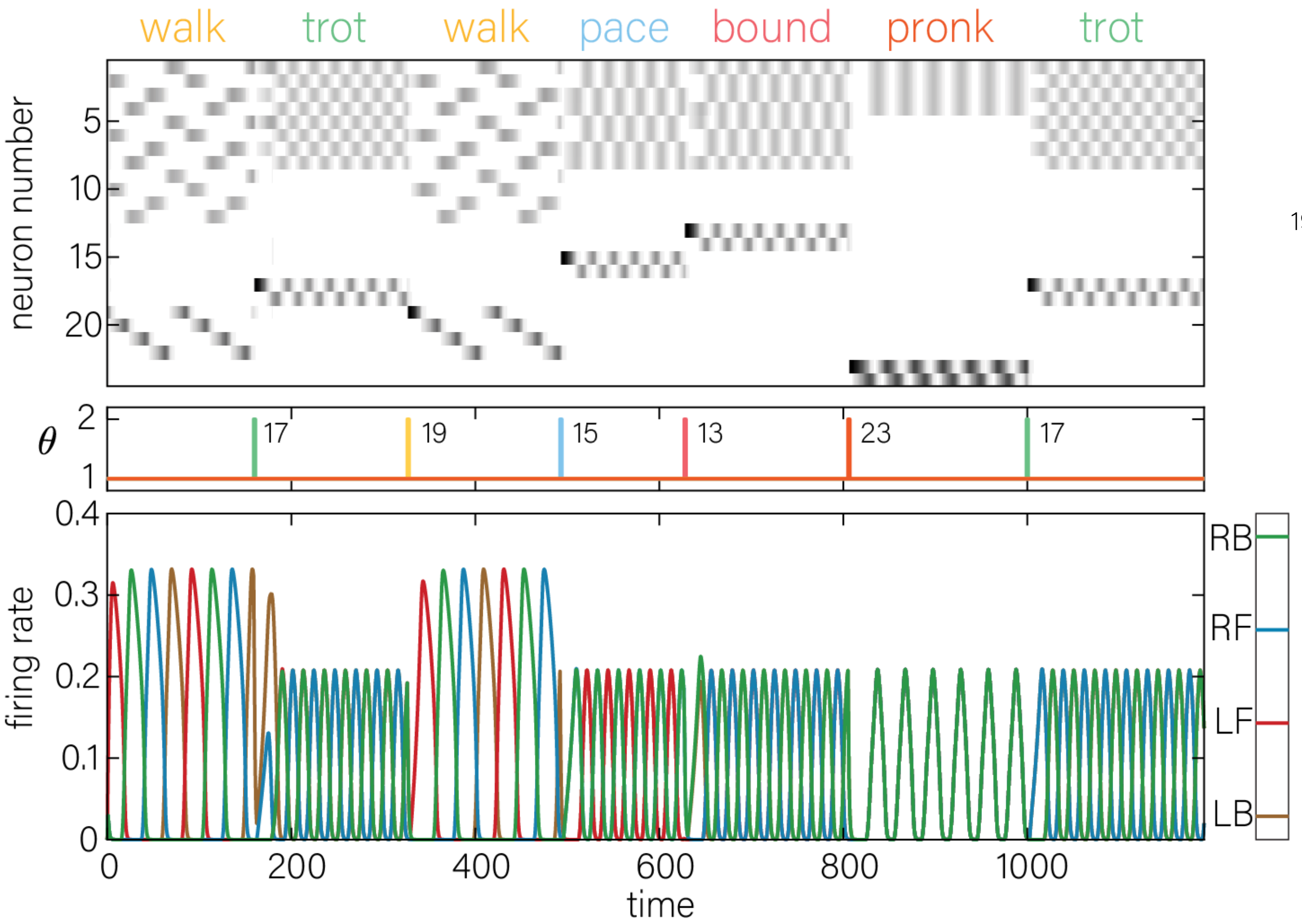


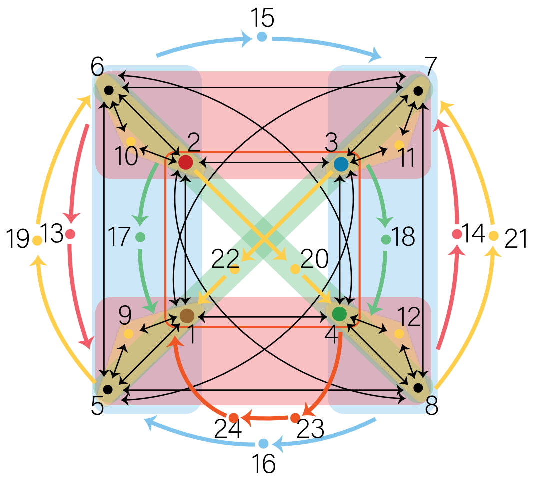
Transiciones de modo de andar: coexistencia.

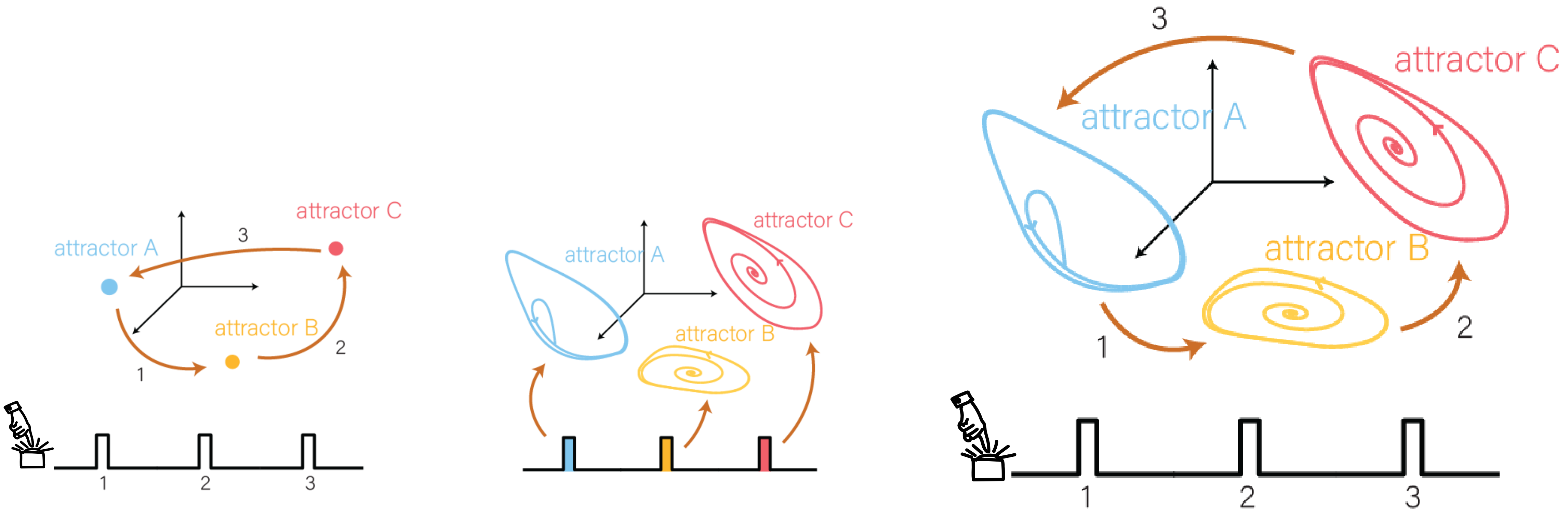
$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

La red puede transicionar entre atractores mediante pulsos enviados a neuronas auxiliares.



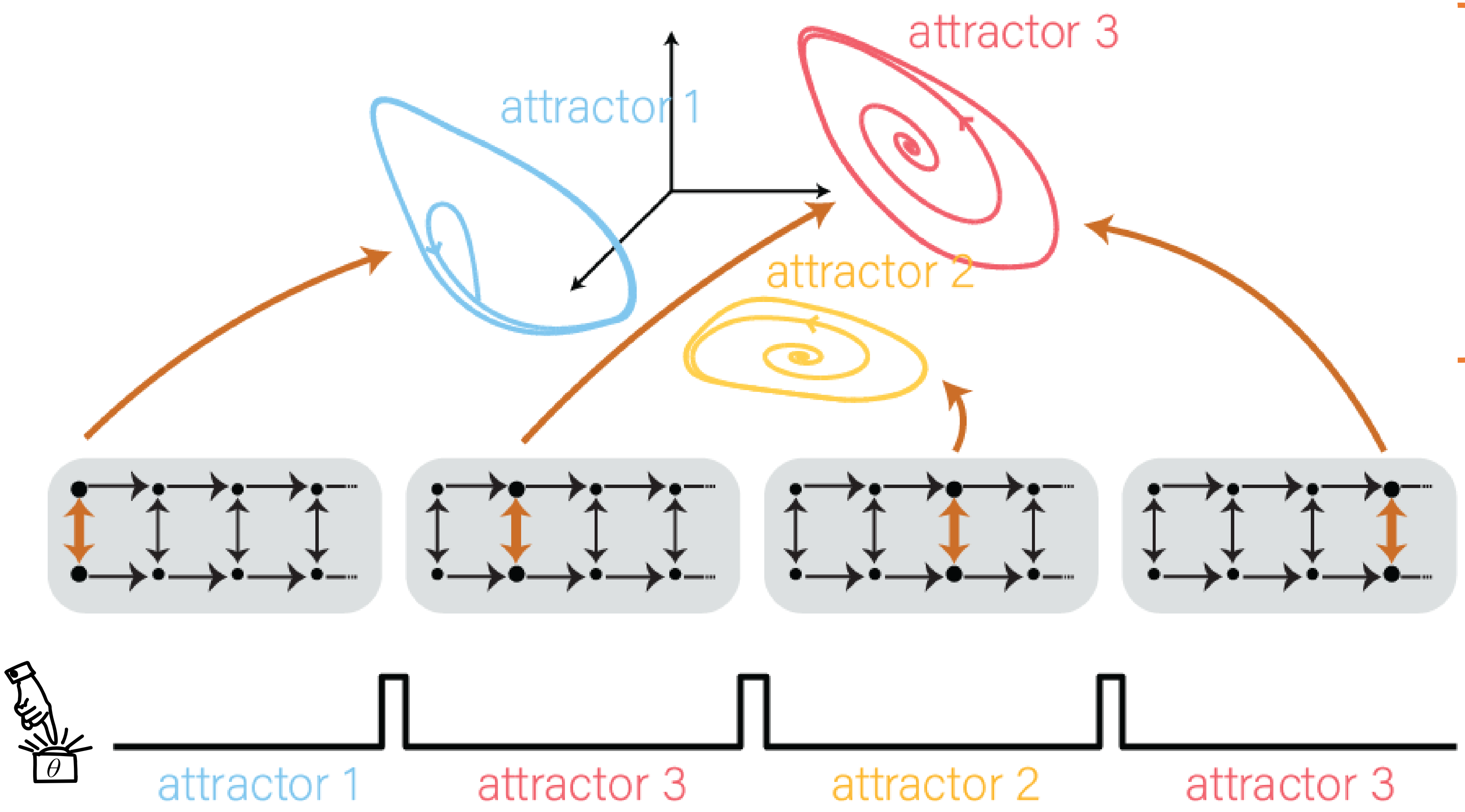






Q3. ¿Pueden múltiples atractores dinámicos ser codificados secuencialmente, accesibles mediante inputs idénticos?

Control secuencial de atractores dinámicos

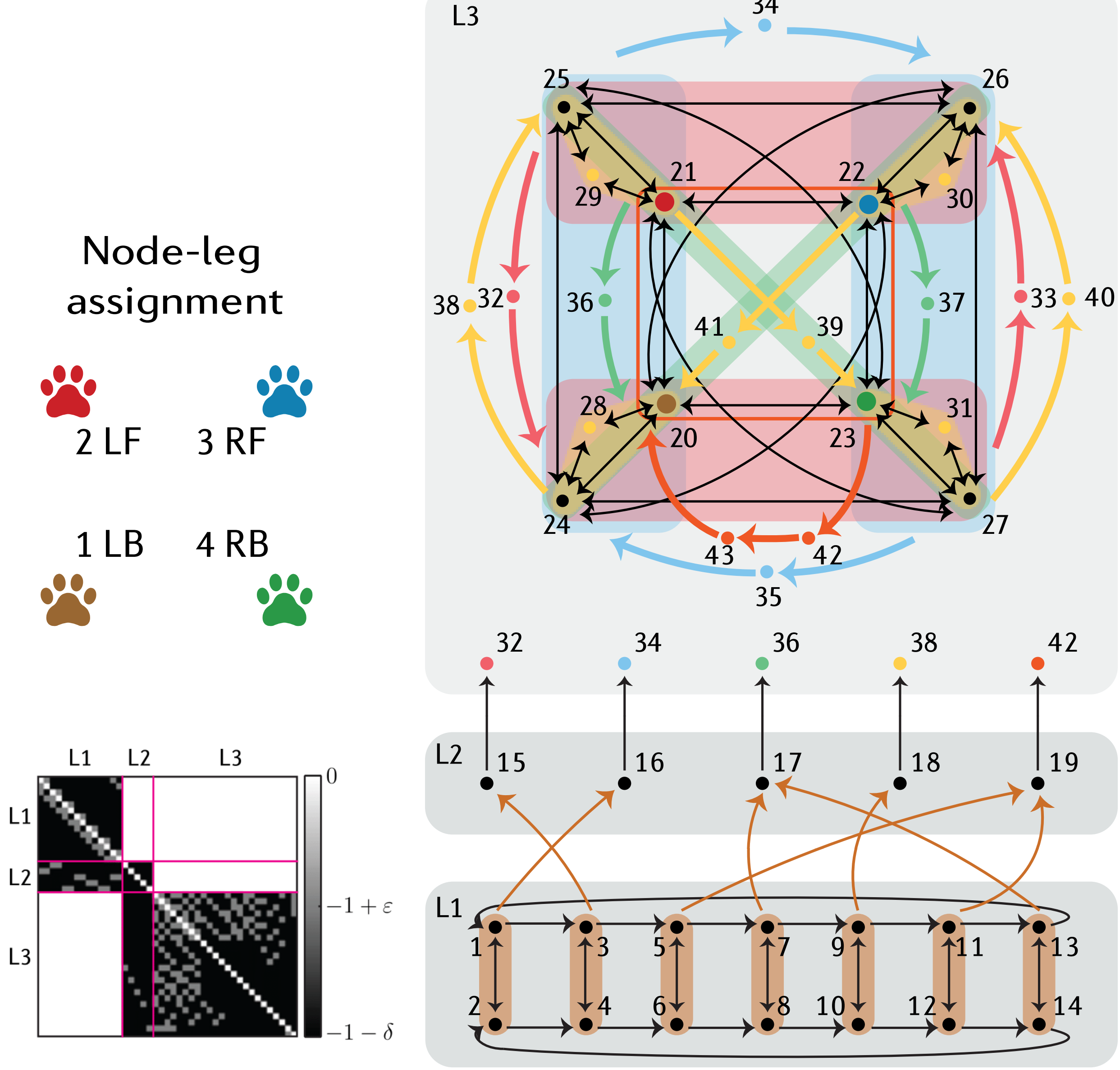


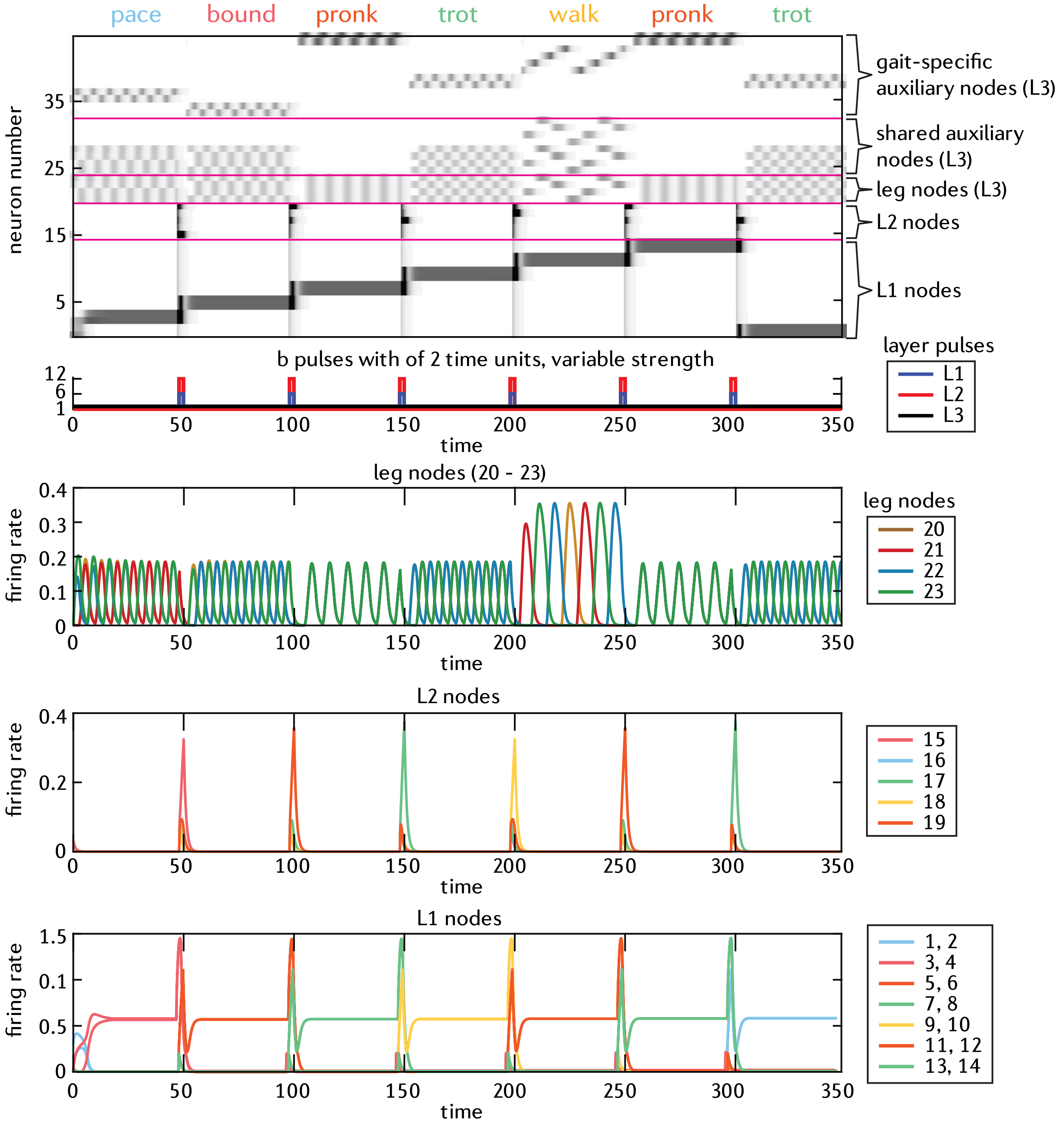
Capa que codifica los commandos motores

Capa que codifica el orden de la sequencia

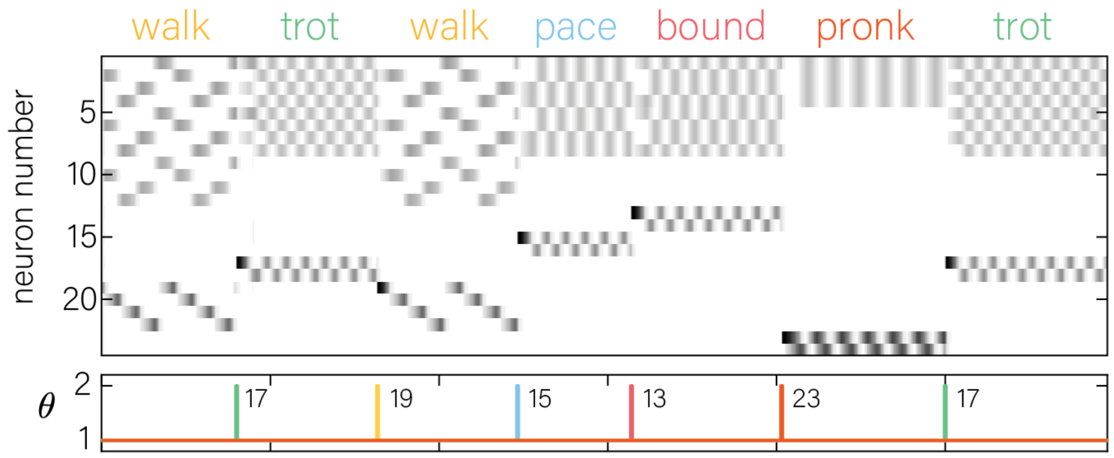
El tiempo es uniforme y controlado externamente

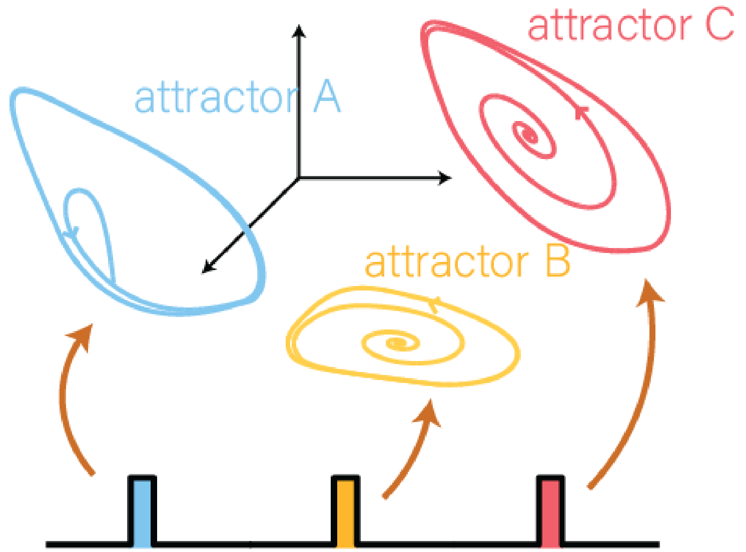
Ejemplo: secuencias de modos de andar



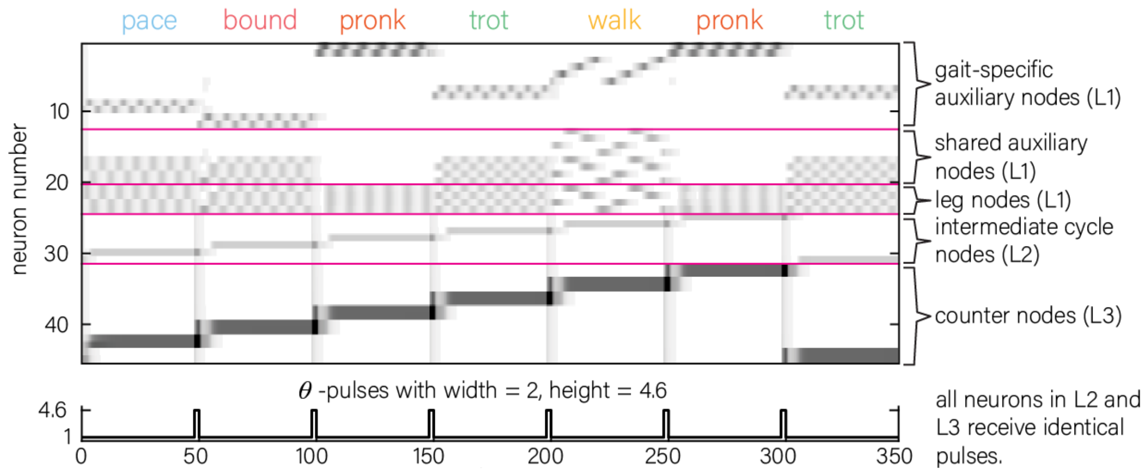


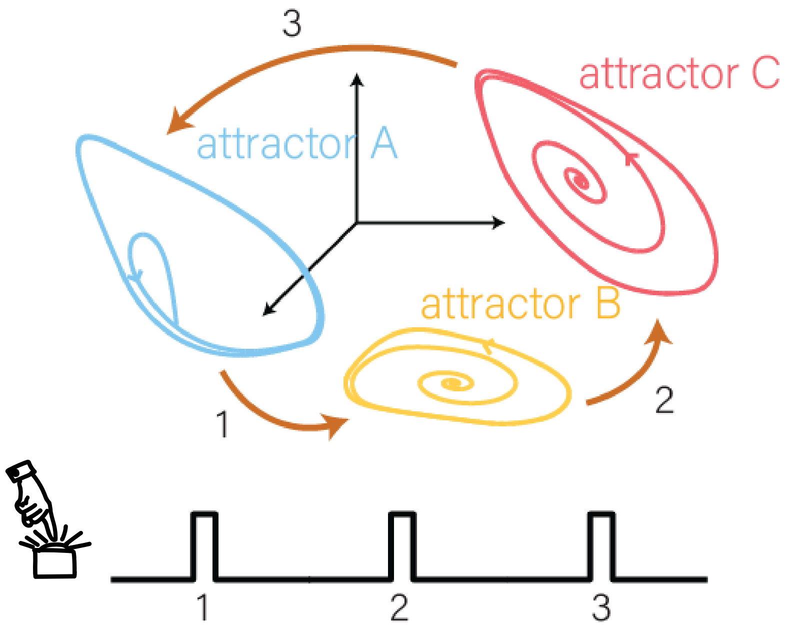
Antes: pulsos específicos, secuencia codificada externamente





Después: pulsos idénticos, secuencia codificada internamente

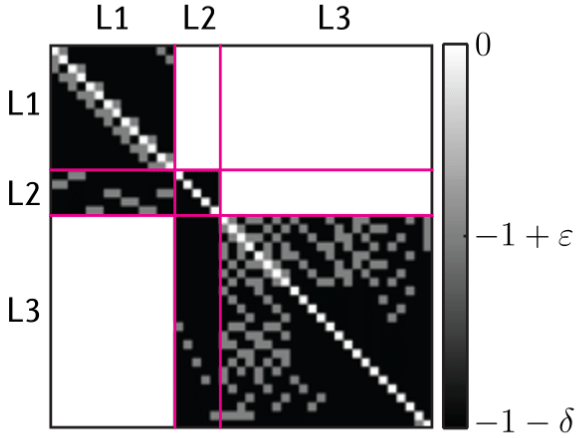


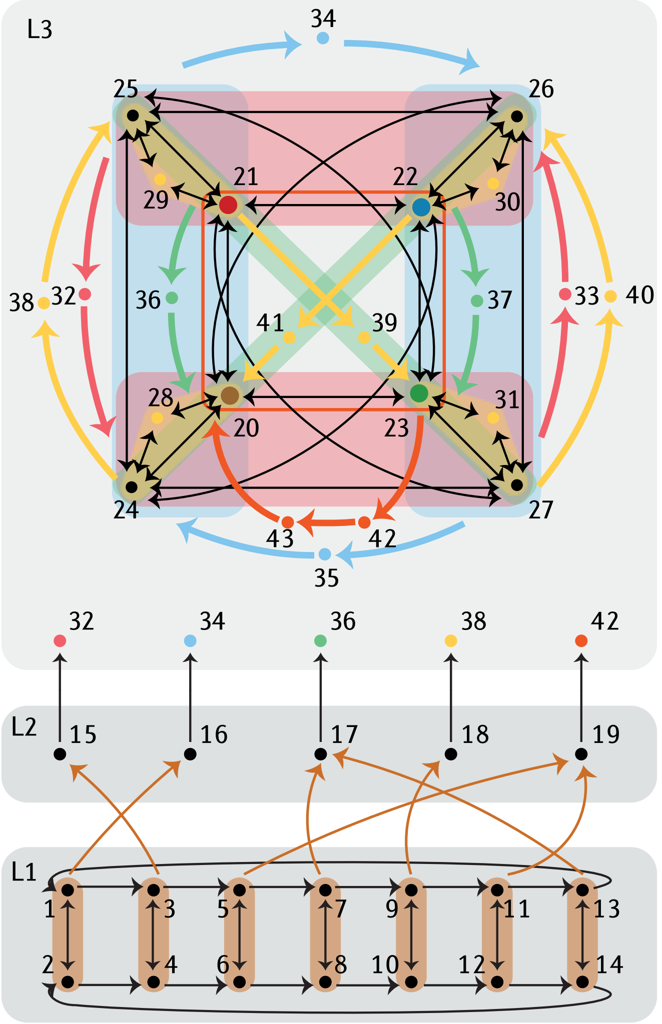


Redes desacopladas están acopladas transitoriamente

Corollary 37 (JLA): Let (W, b) be a competitive non-degenerate TLN. If $b_k \leq 0$ for some $k \in [n]$, then

$$\mathrm{FP}(W, b) = \mathrm{FP}(W_{[n]\setminus\{k\}}, b|_{[n]\setminus\{k\}}).$$





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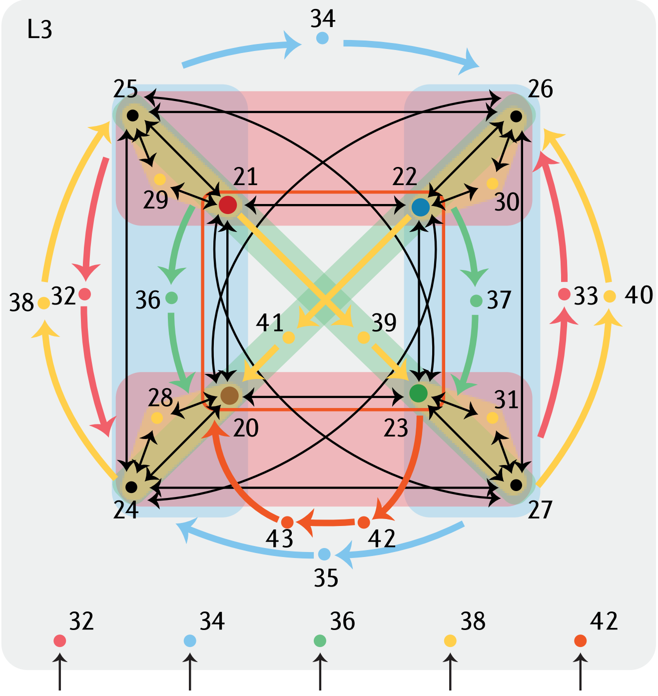
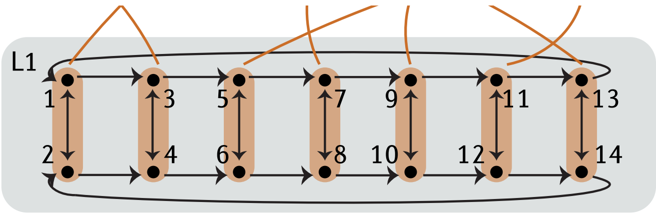
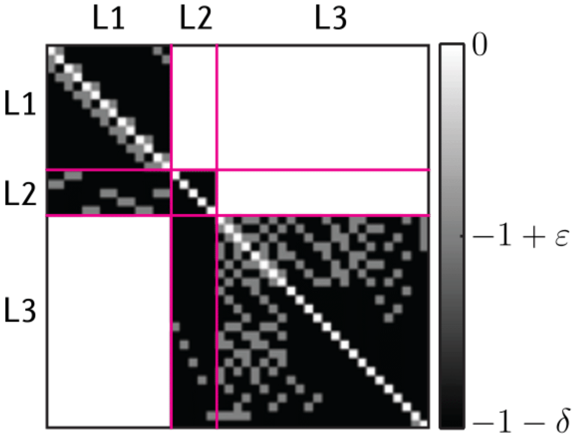


La capa intermedia se desactiva fuera de los pulsos



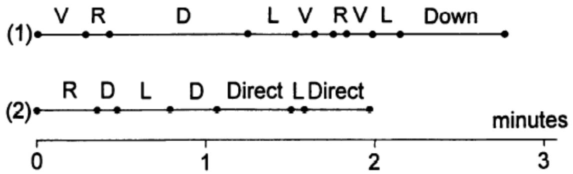


$$W|_{L_1 \cup L_3} = \left[\begin{array}{c|c} W_{L_1} & 0 \\ \hline 0 & W_{L_3} \end{array}\right]$$

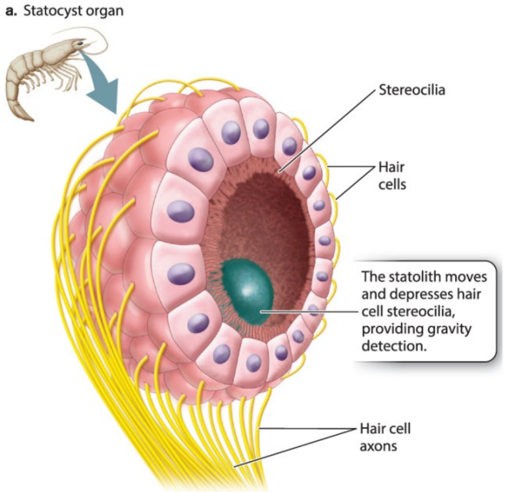
¿Podemos generalizar a otros generadores centrales de patrones?

Ejemplo adicional: el nado de Clione

- Clione carece de sistema visual. ¿Cómo puede cazar? Nadando caóticamente!
- La orientación de Clione es controlada por sus órganos sensoriales gravitacionales: los estatocistos.

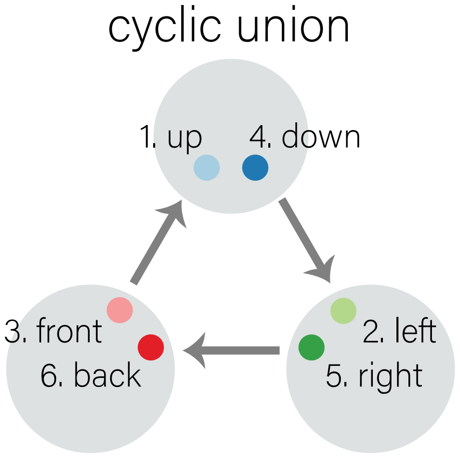


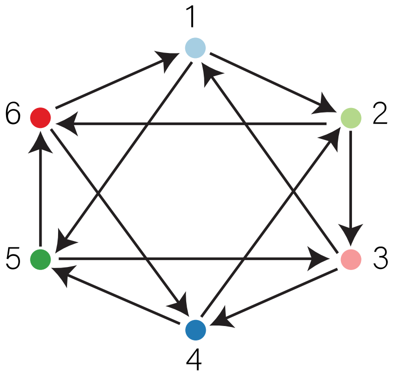
Panchin, Y. V., Arshavsky, Y. I., Deliagina, T. G., Popova, L. B., & Orlovsky, G. N. (1995). Control of locomotion in marine mollusk Clione Limacina. IX. Neuronal mechanisms of spatial orientation. *Journal of neurophysiology*, *73*(5), 1924–1937.

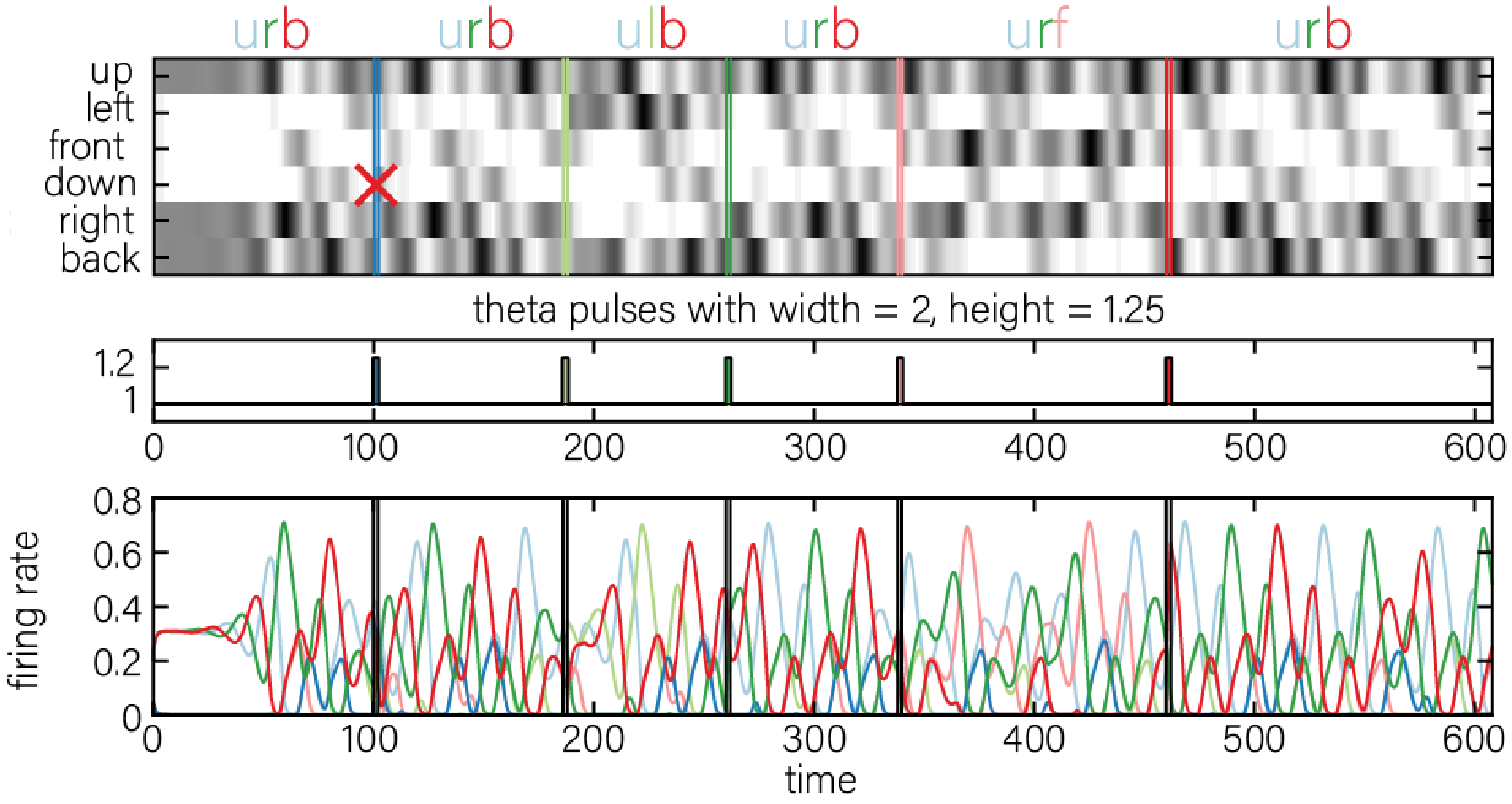


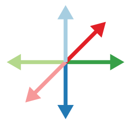
Morris, J. (2019). Biology: How life works.

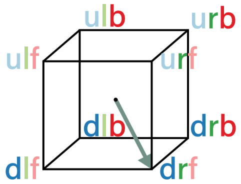
Ejemplo adicional: el nado de Clione





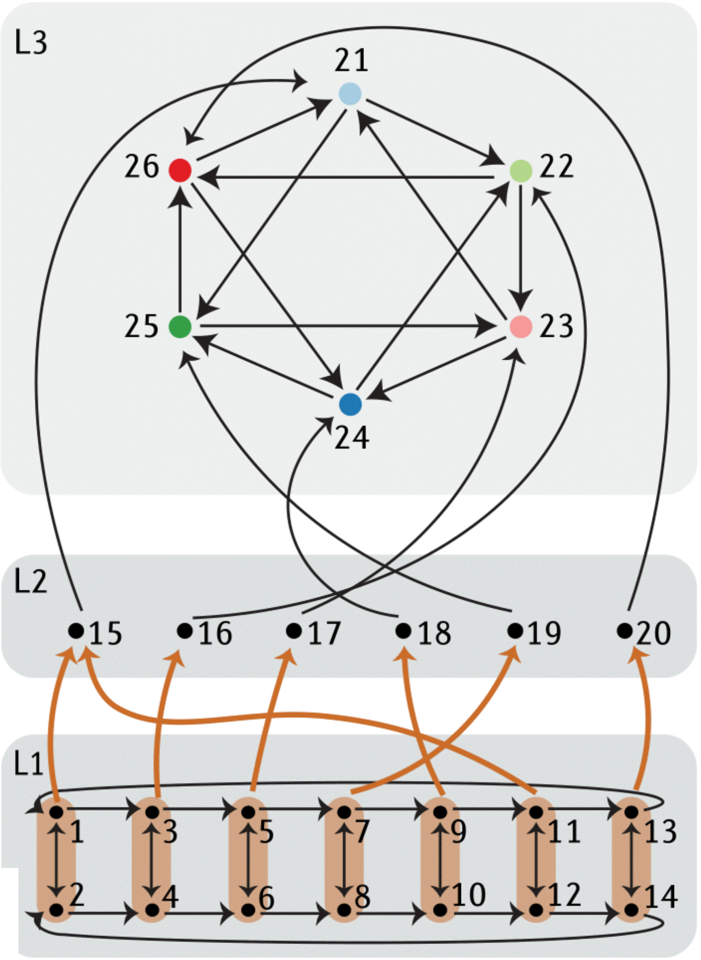


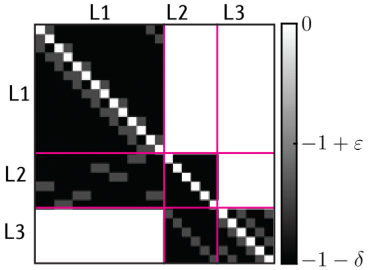


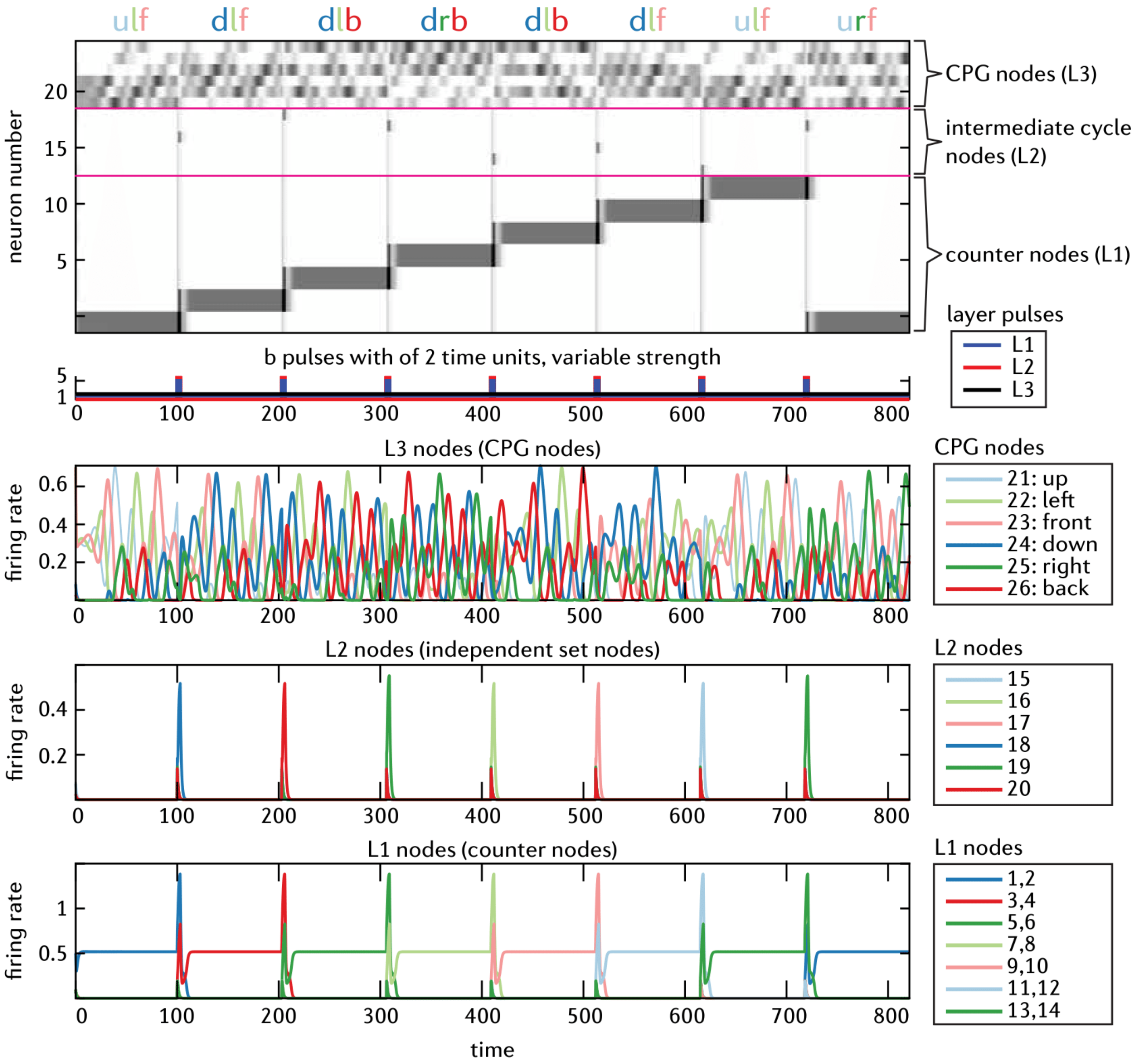


$$|\mathrm{FP}(G)| = 27 \quad \text{minimal supports} = \{\{1,2,3\},\{1,2,6\},\{1,3,5\},\{1,5,6\},\{2,3,4\},\{2,4,6\},\{3,4,5\},\{4,5,6\}\}$$

Ejemplo adicional: el nado de Clione

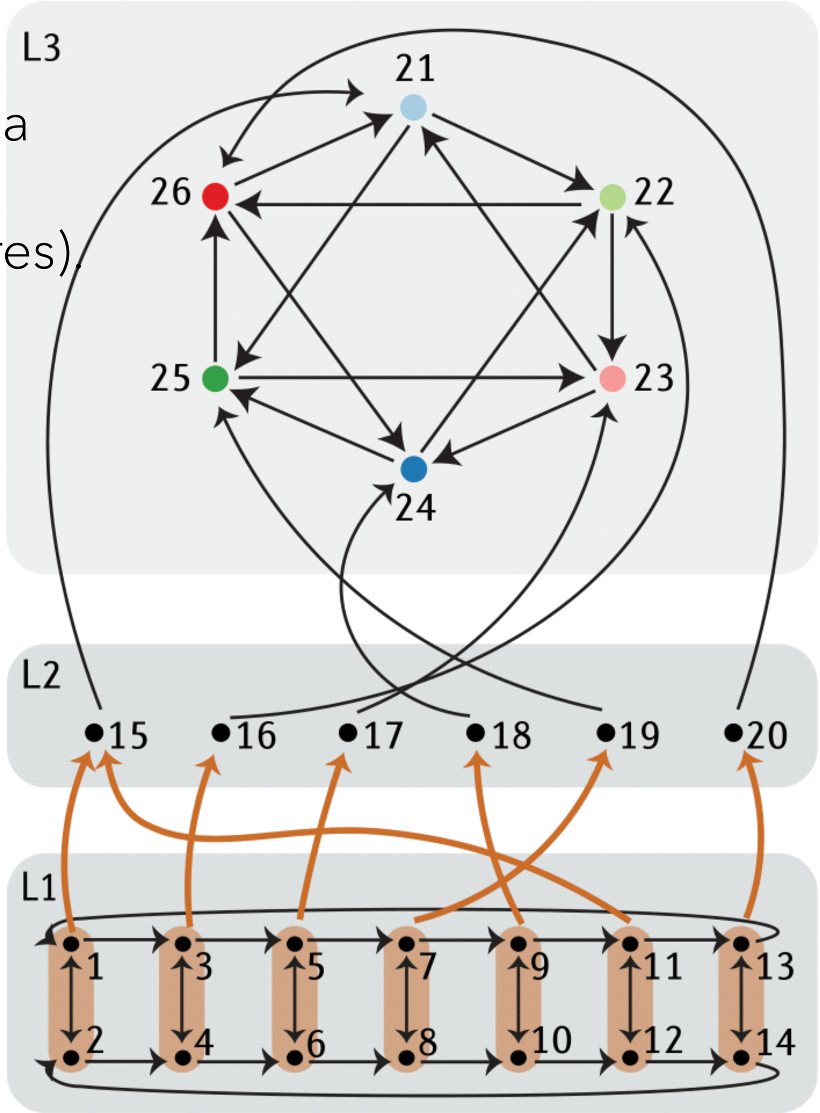






Control secuencial de atractores

Robusto (poca interferencia entre atractores).



Muy pocos parámetros de control, pero aún así mucha flexibilidad.

Secuencias más complejas solo necesitan añadir tres neuronas por acción y un número constante de conexiones.

Gracias!

"They don't appear to want to take over. They just want to dance."

Mahdavian, Navied. The New Yorker, March 2018.

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