Sequence control of quadruped gaits using combinatorial threshold-linear networks

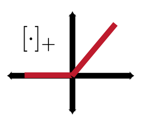
Juliana Londono Alvarez[1], Katherine Morrison[2], Carina Curto[1]

[1]Pennsylvania State University, [2]University of Northern Colorado

How does the brain generate diverse gaits, using a single circuit? How can gaits coexist in a single network? How can we quickly transition between gaits?

How does the brain learn and control motor sequences? How is the order of a sequence encoded in a network?

Combinatorial threshold-linear networks (CTLNs)

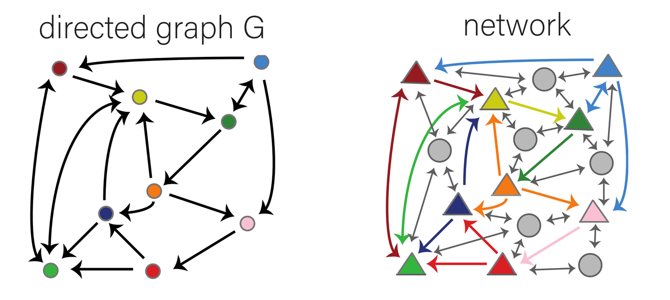


dynamics

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+$$

network model

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } j \to i \text{ in } G, \\ -1-\delta & \text{if } j \not\to i \text{ in } G. \end{cases}$$



$\delta > 0,\ 0 < \varepsilon < \frac{\delta}{\delta+1},\ \theta > 0$

in simulations shown here: $\varepsilon = 0.25, \delta = 0.5, \theta = 0.1$.

Differences in dynamics are due only to differences in the graph G.

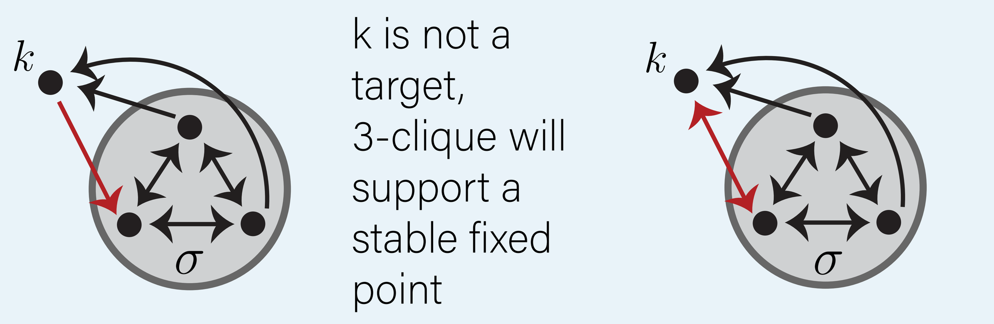
We label all the fixed points of a network by their support: $\sigma = \text{supp}(\mathbf{x}^*) = \{i \mid x_i^* > 0\} \subseteq \{1, \ldots, n\}$

There is at most one fixed point per support, so each fixed point can be identified by its support. The collection of all these supports is: $\text{FP}(G) = \{\sigma \subseteq [n] \mid \sigma \text{ is the support of a CTLN with graph } G\}$

Theorem 1

Theorem: for any graph G, a clique **σ** is the support of a stable fixed point if and only if **σ** is a target-free clique.

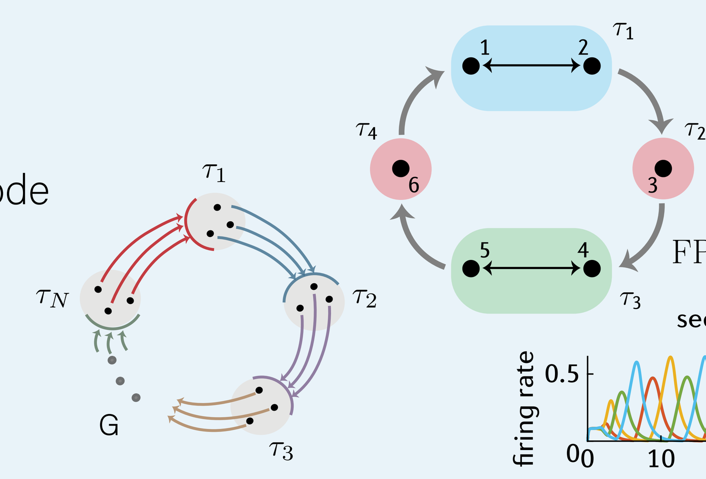
Clique: all to all connected graph.



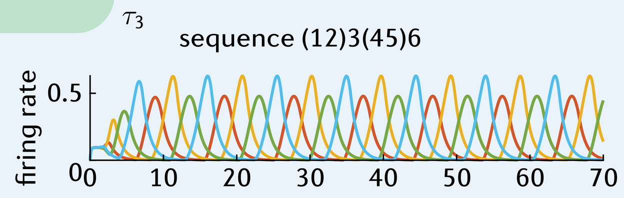
Theorem 2

Cyclic union: edges forward from every node in previous component to every node in next component, and no other edges between components.

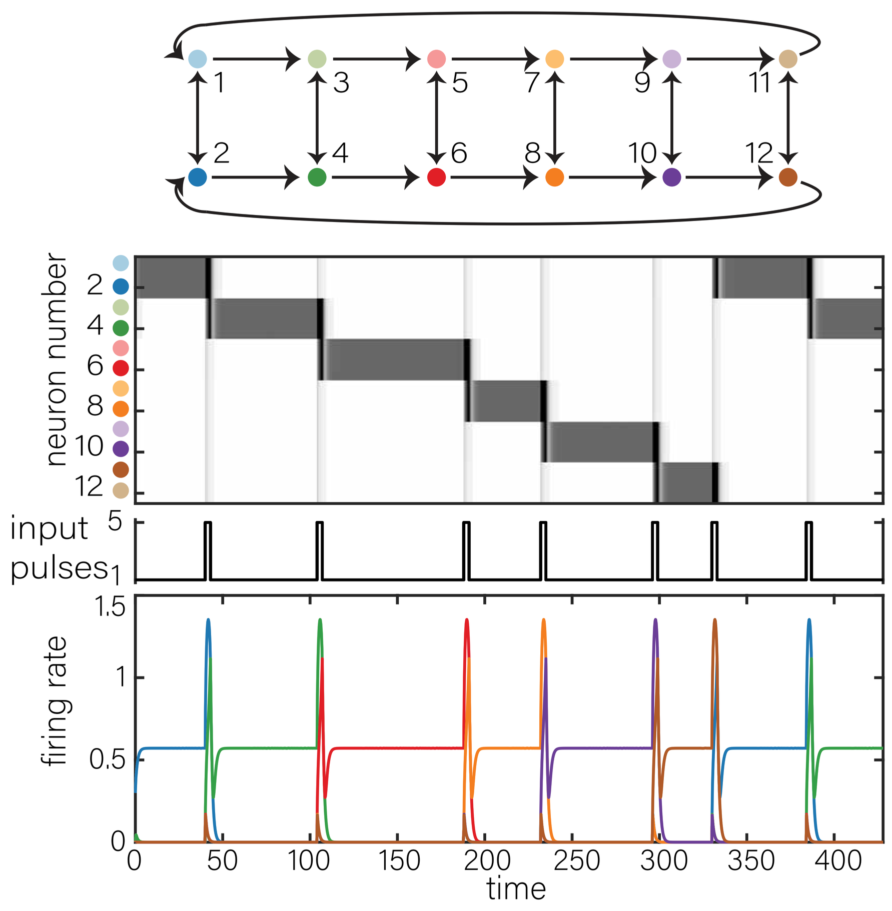
Theorem: for all $i \in [N]$
$\sigma \in \text{FP}(G) \Leftrightarrow \sigma_i \in \text{FP}(G|_{\tau_i})$



$\text{FP}(G|_{\tau_1}) = \{12\}$
$\text{FP}(G|_{\tau_2}) = \{3\}$
$\text{FP}(G|_{\tau_3}) = \{45\}$
$\text{FP}(G|_{\tau_4}) = \{6\}$
$\text{FP}_{\text{core}}(G) = \text{FP}(G) = \{123456\}$

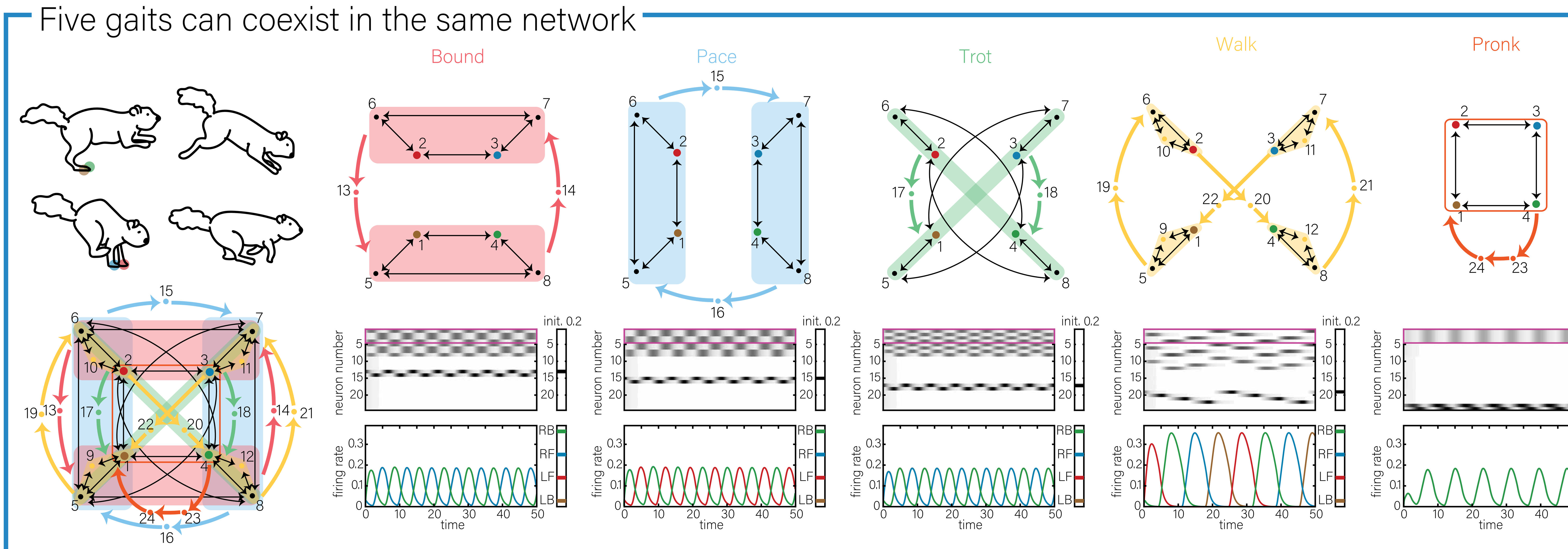


CTLN pulse counter



Every 2-clique is target-free and supports a stable fixed point. Pulses are sent to all neurons in the network. Activity slides to the next clique (stable fixed point) after reception of the pulse. The network keeps track of the number of pulse inputs it has received via the position of the attractor in a linear chain of attractor states (neural integrator model).

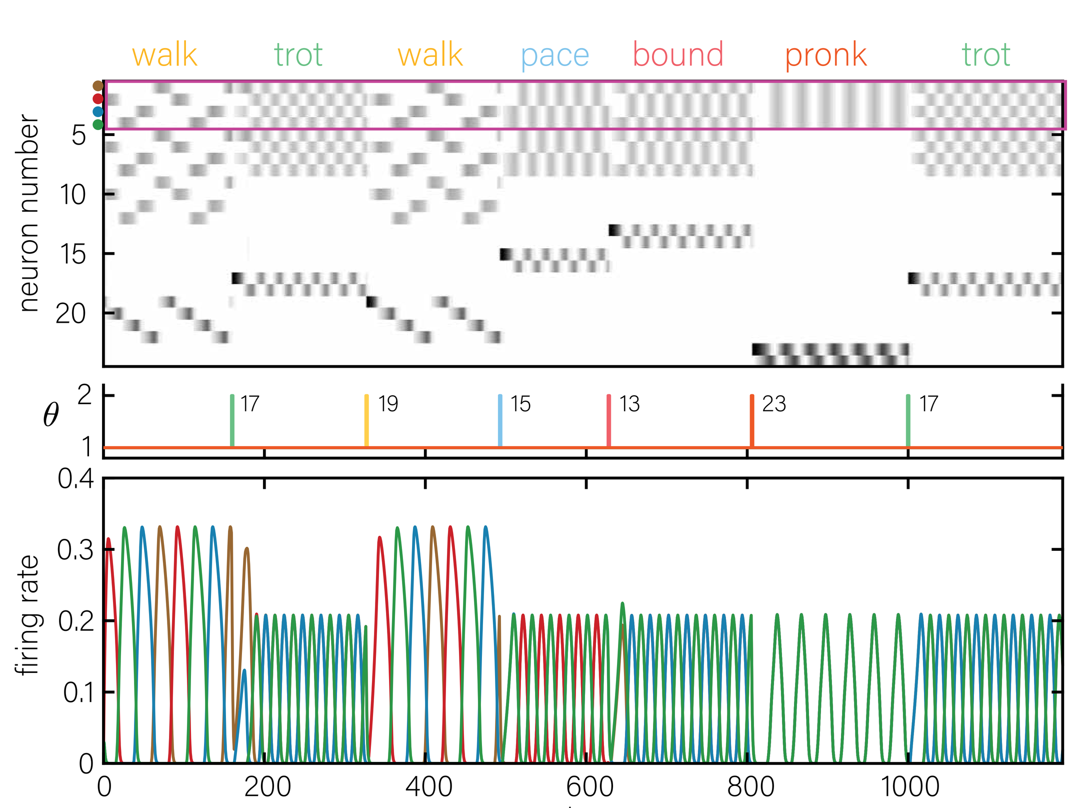
Five gaits can coexist in the same network



All five gaits coexist as distinct limit cycle attractors in the network.

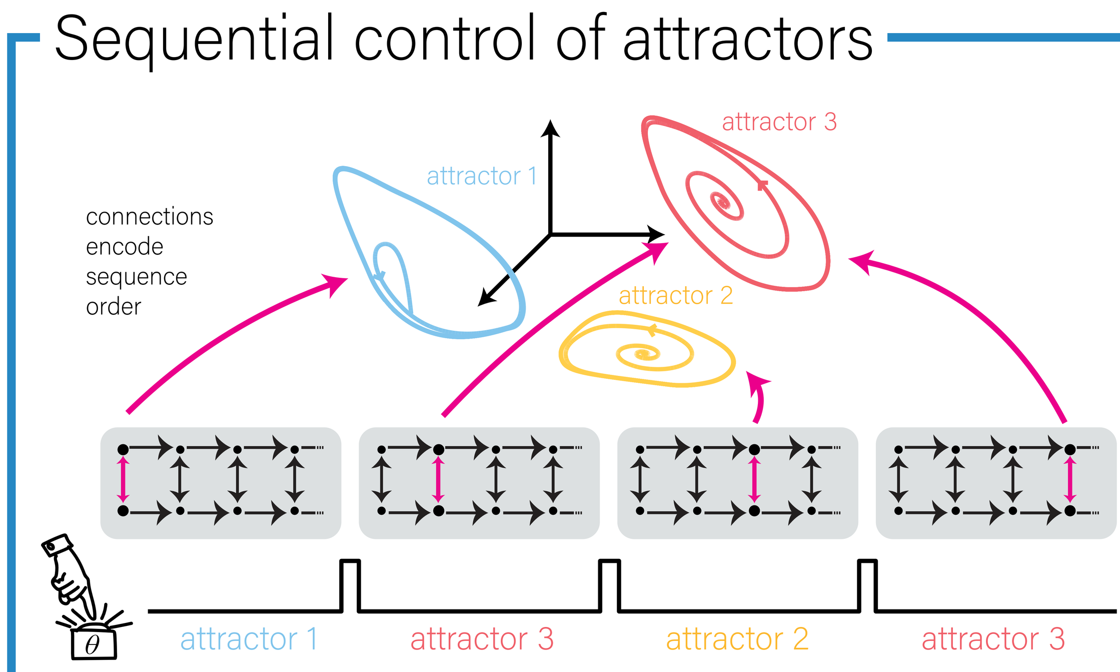
Each gait can be accessed by changes in initial conditons.

Gait transitions



Transitioning between gaits within the network is possible without needing to change parameters. To change gaits, it suffices to stimulate an auxiliary neuron with a pulse. The network quickly settles into the appropriate dynamic attractor corresponding to the gait of the auxiliary neuron stimulated.

Sequential control of attractors

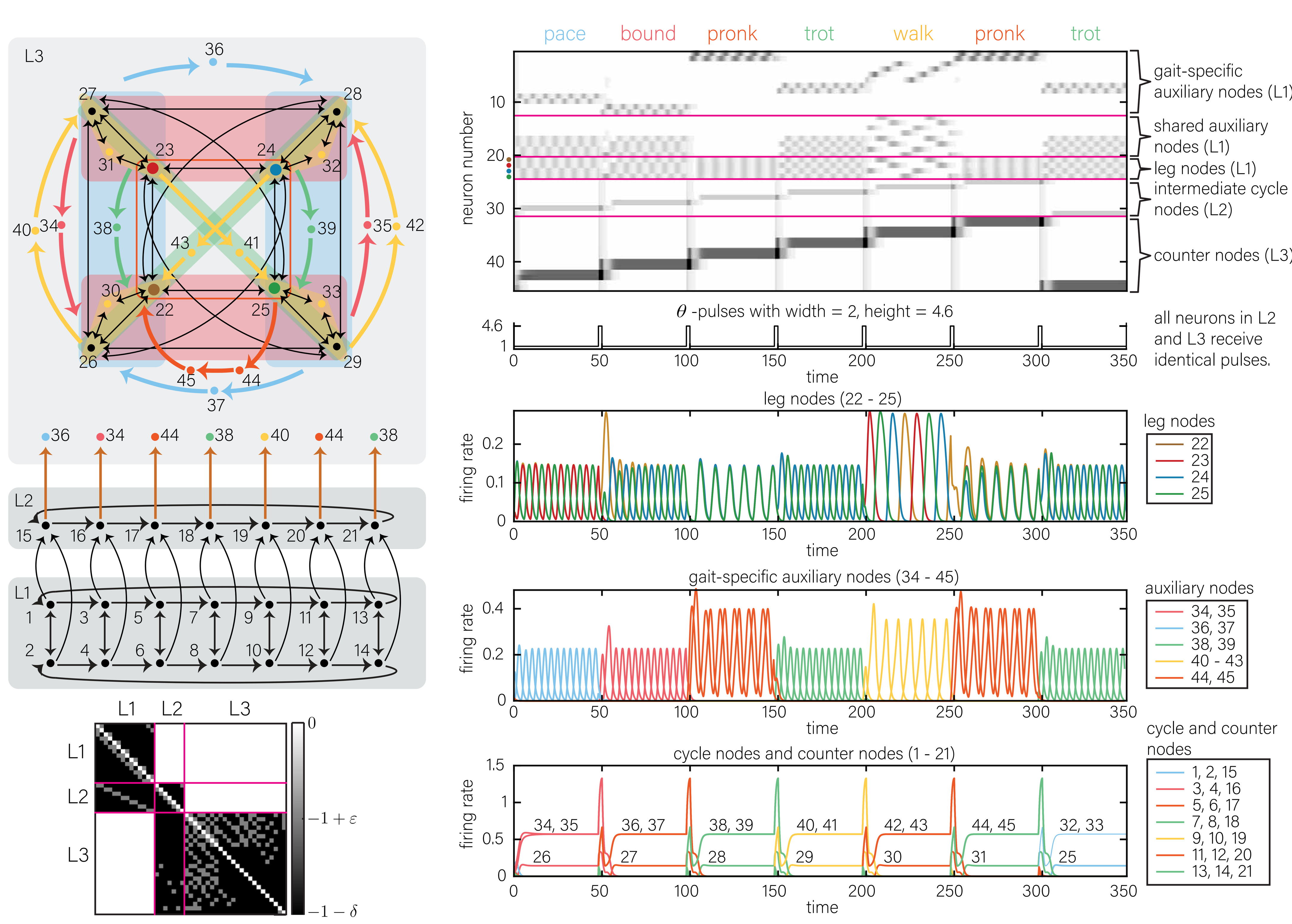


Sequence is encoded separately from attractors.

(Dynamic) attractors are preserved for all possible sequences.

Timing of sequence is dissociated from the sequence. A sequence's rhythm can be altered without affecting attractors or changing parameters.

Layered network for sequential control of quadruped gaits



Gaits will be accessed in the order specified in the connections from L2 to L1 (auxiliary nodes corresponding to different gaits redrawn at the bottom of L1 for clarity). Only neurons in L2 and L3 receive pulses, and so the sequence is stored within the network. Each pulse moves L2 and L3 one step to the right, activating the next gait down the sequence. Cycle nodes and counter nodes are one step out of synch with the gait sequence. Any sequence can be encoded between layers L2 and L3 and layer L1.

References:

- C. Curto, J. Geneson and K. Morrison. Fixed points of competitive threshold-linear networks. Neural Computation, 31, 1, 94-155, 2019.
- K. Morrison and C. Curto. Predicting neural network dynamics via graphical analysis. Chapter in Algebraic and Combinatorial Computational Biology. R. Robeva and M. Macaulay (Eds) 2018.
- Parmelee C, Londono Alvarez J, Curto C and Morrison K. Sequential attractors in combinatorial threshold-linear networks. SIAM Journal on Applied Dynamical Systems 2022 21:2, 1597-1630.
- Golubitsky M, Stewart I, Buono P, Collins J. A modular network for legged locomotion. Physica D. 1998, 115, 56-72.
- Gambarian P. P. How Mammals Run: Anatomical Adaptations. Wiley 1974.

Funding: NIH R01 NS120581, NSF DMS-1951165, NIH R01 EB022862, NSF DMS-1951599.